Advanced OFDM Modulators considered in the IST-WINNER framework for future wireless systems

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Abstract—The goal of this contribution is twofold: i) an overview on the vast literature on recent advances related to OFDM modulators is given: Cyclic-Prefix OFDM, Zero-Padded OFDM, Pseudo-Random-Postfix OFDM and IOTA-OFDM; these are identified to be potential candidates for fourth generation mobile communication networks in the framework of the IST-WINNER project. ii) The advantages/disadvantages of the given OFDM derivates are discussed in terms of spectral efficiency, robustness to frequency selective fading and suitability to mobility scenarios requiring channel tracking. Simulation results are shown in order to illustrate differences in terms of robustness to fading and mobility contexts. As a conclusion, it is shown that the choice of the optimum modulation scheme depends on the considered scenario and in particular on desired trade-offs in terms of throughput increase (IOTA-OFDM) and simplicity of channel tracking without pilot overhead (PRP-OFDM).

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing seems to be the preferred modulation scheme for many modern broadband communication systems. Its inherent robustness to multi-path propagation and appealing low complexity equalization properties have already made it a candidate either for high speed modems over twisted pair (digital subscriber lines xDSL), terrestrial digital broadcasting (Digital Audio and Video Broadcasting: DAB) and 5GHz Wireless Local Area Networks (WLAN: IEEE802.11a [1], IST-BROADWAY [2] and the future IEEE802.11n).

In the framework of the IST-WINNER project, it is intended to extend the use of OFDM to future wide-range mobile communication systems. This context motivates the comparison of recent evolutions of the standard Cyclic Prefix OFDM (CP-OFDM) modulation scheme. Indeed, modulators like Zero-Padded OFDM (ZP-OFDM) allow the implementation of more robust receiver architectures; Pseudo-Random-Postfix OFDM (PRP-OFDM) combines these advantages with the possibility to estimate and track the channel impulse response (CIR) blindly without any loss in throughput nor spectral efficiency compared to CP-OFDM. IOTA-OFDM finally proposes means to reduce the symbol overhead, since it does not require any prefix- nor postfix sequence.

This paper puts these recent advances in a common framework, discusses advantages and disadvantages and gives a performance analysis by means of Monte Carlo simulations.

The structure is as follows: section II introduces the standard CP-OFDM modulation scheme and defines all notations used throughout this paper. III extends the definitions to the context of ZP-OFDM modulators; a corresponding discussion for PRP-OFDM is presented in section IV and IOTA-OFDM is analyzed in section V. Simulation results and a final conclusion follow in section VI and VII respectively.

II. NOTATIONS AND CP-OFDM MODULATOR

This section defines the analogue CP-OFDM signal and its generation by a generic synthesis filter bank approach. Then, the corresponding discrete transceiver representation and a discrete channel model are given. Based on these definitions, the following sections will detail and discuss the differences of ZP-OFDM, PRP-OFDM and IOTA-OFDM compared to standard CP-OFDM schemes.

A. Synthesis filter bank: the transmitted signal

Define the constellation symbols to be transmitted as $s_n(k)^2$, $n = 0, \ldots, N-1$, $k \in \mathbb{Z}$. They are modulated onto $N$ parallel and distinct sub-channels by shaping filters $g_n(t)$, $n = 0, \ldots, N-1$ which form an orthogonal basis [3]. The useful symbol duration is denoted $T_s$. A cyclic prefix of length $T_{cp}$ usually extends this symbol duration to avoid Inter-Block-Interference (IBI). We note $T_B = T_u + T_{cp}$ the total symbol duration and $g_n'(t)$ the corresponding extended functions.

The transmitted time domain signal $u(t)$ of the CP-OFDM modulator is the sum of the filtered data symbols:

$$u(t) = \sum_{k \in \mathbb{Z}} \sum_{n=0}^{N-1} s_n(k) g_n'(t - kT_B)$$

where $T_B$ is the duration of one OFDM symbol block. In traditional OFDM systems, the filters $g_n(t)$ and their extended versions $g_n'(t)$ are chosen to be

$$g_n(t) = \frac{1}{\sqrt{T_u}} \text{Rect}_t(e^{2\pi j n t/T_u})$$

$$g_n'(t) = \frac{1}{\sqrt{T_u}} \text{Rect}_t(e^{2\pi j n(t-T)} T_u)$$

where $\text{Rect}_T(t)$ is the window function of duration $T$.

$$\text{Rect}_T(t) = \begin{cases} 1 & 0 \leq t < T \\ 0 & \text{otherwise.} \end{cases}$$

1This work has been performed in the framework of the IST project IST-2003-507581 WINNER, which is partly funded by the European Union. The authors would like to acknowledge the contributions of their colleagues.
It is straightforward from (2) and (3) to prove that the $g_{\nu}^r(t)$ and $g_{\nu}(t)$ function basis meet the orthogonality constraints for $0 \leq \Delta T \leq T_{cp}$:

$$
\int_{-\infty}^{\infty} g_{\nu}^r(t-kT_B)g_{\nu}'(t-k'T_B-\Delta T)\,dt = \delta_{n,n'}\delta_{k,k'}e^{j2\pi n'(\Delta T-T_{cp})},
$$

where $\delta_{k,k'}$ is the Kronecker delta defined as

$$
\delta_{i,j} = \begin{cases} 1 & \text{for } i = j; \\ 0 & \text{otherwise}. \end{cases}
$$

B. The discrete transceiver model

The baseband discrete-time block equivalent model of an $N$ carrier CP-OFDM system is considered. The $k^{th}$ $N \times 1$ input digital vector $\tilde{s}_N(k)$ is first modulated by the IFFT matrix $F_N^H = \frac{1}{\sqrt{N}} W_N^{-mT}, 0 \leq n < N, 0 \leq m < N$ and $W_N = e^{-j2\pi \frac{n}{N}}$. Then, the guard interval is added by cyclically repeating the last $D$ samples prior to the data symbol. With

$$
F_{cp}^H = \begin{bmatrix} 0_{N-D} & I_D \end{bmatrix}_{P \times N} F_N^H,
$$

the transmitted symbol is

$$
\tilde{s}^H(k) = F_{cp}^H \tilde{s}_N(k),
$$

(5)

The role of the cyclic prefix is to turn the linear convolution into a set of parallel attenuations in the discrete frequency domain. $\tilde{s}^H(k)$ is then sent sequentially through the channel modeled here as an FIR filter of order $L$, $H(z) = \sum_{n=0}^{L} h_n z^{-n}$. The OFDM system is designed such that the postix duration exceeds the channel memory $L \leq D$. Let $H_{ISI}(N)$ and $H_{IBI}(N)$ be respectively the size $N$ Toeplitz inferior and superior triangular matrices of first column $[h_0, h_1, \ldots, h_{L-1}, 0, \ldots, 0]^T$ and first row $[0, 0, \ldots, 0, h_1, \ldots, h_L]$. As already explained in [3], after guard interval suppression the received signal can be expressed as $r_N(k) = H_{ISI}(N)\tilde{s}_N(k) + n_N(k)$; here, $H_{ISI}(N) = H_{ISI}(N) + H_{IBI}(N)$ is a circular matrix that is diagonalized on a Fourier basis and $H_{ISI}(N)$ and $H_{IBI}(N)$ represent respectively the intra and inter block interference. $n_N(k)$ is the $k$th AWGN vector of element variance $\sigma^2_n$. Corresponding low complexity equalization schemes are presented in [3]. In the context of a coherent modulation, the receiver requires an estimate of the CIR; this is typically obtained by learning symbols known to both, the transmitter and the receiver. In the presence of mobility, pilot symbols are introduced in order to facilitate the tracking of the CIR [4]. In the following sections, it will be shown that improved decoding performances can be obtained with the so-called Zero-Padded OFDM (ZP-OFDM) modulation scheme - still requiring learning symbols and/or pilot tones in order to estimate and track the CIR. Pseudo-Random-Postix OFDM is presented afterwards - this modulation scheme is shown to keep the advantages of ZP-OFDM and additionally provides means to estimate and track the CIR without the typical overhead introduced by pilot tones.

III. ZP-OFDM MODULATOR

The ZP-OFDM modulation scheme has been studied in [5], [6]. The idea is to replace the guard interval sequence of CP-OFDM by a zero padding sequence of identical length. The corresponding continuous modulator corresponds to (1) with the difference that the duration of the window function (4) is limited to the duration of the OFDM symbol duration without any prefix/postix sequence.

The corresponding discrete modulator is obtained by replacing $F_{cp}^H$ in (5) by $F_{zp}^H$:

$$
F_{zp}^H = \begin{bmatrix} I_N \end{bmatrix}_{P \times N} F_N^H.
$$

With the same transmission scheme, several different equalization approaches are possible in the receiver [5] ranging from low complexity/medium performance (Overlap-Add based) to high complexity/high performance (MMSE based equalization). In particular, [5] demonstrates that (contrarily to CP-OFDM) symbol equalization is possible even if frequency domain channel nulls fall onto data carriers. However, ZP-OFDM still requires channel estimates which are typically obtained by learning symbols and/or pilot tones.

IV. PRP-OFDM MODULATOR

The PRP-OFDM modulation scheme has been studied in [7] for the single-antenna context and in [8] for the multiple-antennas context. The idea is replace the zero-sequence of ZP-OFDM by a pseudo-randomly weighted deterministic sequence known to both, the transmitter and the receiver.

The corresponding continuous modulator is derived as follows: the transmitted time domain signal $u(t)$ of the PRP-OFDM modulator is the sum of the filtered data symbols plus the postix sequence $p(t) = \sum_{k \in \mathbb{Z}} \alpha(k)p_s(t - kT_B)$:

$$
u(t) = \sum_{k \in \mathbb{Z}} \left[ \alpha(k)p_s(t - kT_B) + \sum_{n=0}^{N-1} \tilde{s}_N(k)g_n(t - kT_B) \right]
$$

Typically, $p_s(t) = 0$ for all $t$ with $g_n(t) \neq 0$ and vice versa. The block duration of a PRP-OFDM symbol including its postix is defined to be $T_{cp}$. PRP-OFDM symbol block $k$ has one postix $p_s(t)$ attributed to it; it is weighted by a pseudo-random scalar $\alpha(k) \in \mathbb{C}$ known to both the transmitter and the receiver [9]. In the framework of this paper, all $\alpha(k)$ are assumed to be a pure phase, i.e. $\alpha(k) = e^{j\phi(k)}$. These weighting factors prevent any signal stationarity and thus spectral peaks. This is desirable, since in particular in the presence of frequency selective fading the spectrum should be as flat as possible - otherwise, the system performance would strongly depend on the band affected by the fading.

As illustrated in Fig. 1, the corresponding discrete modulator is obtained by adding the pseudo randomly weighted postix sequence to the ZP-OFDM modulator outputs:

$$
s_r(k) = F_{zp}^H \tilde{s}_N(k) + \alpha(k)c_p
$$

with $P = N + D$, $c_p = (0_{1,N} c_D)^T$ and $c_D$ contains the postix sequence samples. The expression of the received block is thus:

$$
r_r(k) = H_{pk} \left( F_{zp}^H \tilde{s}_N(k) + \alpha(k)c_p \right) + n_r(k)
$$

$$
r_r(k) = H_{pk} \left( F_{zp}^H \tilde{s}_N(k) + \alpha(k)c_D \right) + n_r(k)
$$

Hereby, $H_{pk} = H_{ISI}(P) + b_k H_{IBI}(P)$ and $b_k = \frac{\alpha(k)}{\alpha(k-1)}$. Note that $H_{pk}$ is diagonalized on a new basis which is different, but still similar to the Fourier basis [7]. The actual choice of the postix sequence is discussed in [9].
ZP-OFDM: different equalization approaches are possible in the receiver ranging from low complexity/medium performance (Overlap-Add based) to high complexity/high performance (MMSE based equalization). Additionally to these features, PRP-OFDM allows simple and higher. Since the postx sequences are of same power and mean-value calculation is sufficient to extract the postx sequence: A first idea consists in exploiting that the OFDM duration as the guard interval of CP-OFDM, a higher spectral efficiency can be obtained; in particular, the typical overhead in terms of learning symbols and pilot tones for CP-OFDM is avoided.

V. IOTA-OFDM MODULATOR

A. Generalities on OFDM/QAM

OFDM/OffsetQAM is an alternative to conventional OFDM modulation. Contrary to it, OFDM/QAM modulation does not require the use of a guard interval, which leads to a gain in spectral efficiency. Although a guard interval is a simple and efficient way to combat multi-path effects, better performance can be reached by modulating each subcarrier by a prototype function [14], [15]. To obtain the same robustness to the multi-path effects as OFDM with a guard interval, this prototype function must be very well localized in both the time and frequency domains. The localization in time aims at limiting the inter-symbol interference and the localization in frequency aims at limiting the inter-carrier interference (e.g. due to Doppler effects).

The orthogonality between the sub-carriers must also be maintained after the modulation. Optimally localized functions having these properties exist but they guarantee orthogonality for real valued symbols only. An OFDM modulator using these functions is denoted OFDM/OffsetQAM. We can note that in OFDM/QAM, each sub-carrier carries a real valued symbol but the density of the sub-carriers in the time-frequency plane is two times greater than in conventional OFDM, also called OFDM/QAM, with no guard interval. This means \( \tau_0 v_0 = 1/2 \), where \( \tau_0 \) is the OFDM/QAM symbol duration and \( v_0 \) denotes the inter-carrier spacing (see Fig. 3). Thus, OFDM/QAM has the same spectral efficiency as conventional OFDM with no guard interval. The transmitted signal can be expressed as follows

\[
s(t) = \sum_{k \in \mathbb{Z}} \sum_{n=0}^{N-1} a_{n,k} e^{j(\pi/2) e^{j\pi v_0 t}} g(t - k \tau_0)
\]

where \( N \) is the number of sub-carriers, \( a_{n,k} \) is the real valued symbol transmitted on the \( n^{th} \) sub-carrier at the \( k^{th} \) symbol; \( g(t) \) denotes the real valued prototype function. (6) can be rewritten

\[
3 \tau_0 = T_u / 2, \ T_u \text{ being the useful part of the conventional OFDM symbol duration}
\]
In a simpler manner as

\[ s(t) = \sum_{k \in \mathbb{Z}} \sum_{n=0}^{N-1} a_{n,k} g_{n,k}(t) \]  

(7)

where \( g_{n,k}(t) \) are the shifted versions of \( g(t) \) in time and frequency. Therefore the orthogonality condition among the sub-carriers is

\[ \Re \left( \int_{-\infty}^{\infty} g_{n,k}(t) g_{n',k'}(t) dt \right) = \delta_{n,n'} \delta_{k,k'} \]  

(8)

In discrete time, the rewriting of (7) leads to

\[ s[i] = \sum_{k \in \mathbb{Z}} \sum_{n=0}^{N-1} a_{n,k} g_{n,k}[i] \]

with \( g_{n,k}[i] = g[i - kN/2]e^{j(n+k)\pi/2^2/2 \pi / N \cdot i} \). Demodulation is performed by applying the real scalar product on the orthogonal basis of functions.

\[ \hat{a}_{n,k} = \Re \left( \sum_{i} g_{n,k}^*[i] s[i] \right) \]

A discrete implementation of an IOTA-OFDM modem using filter-banks is given by Fig. 2.

**B. The IOTA function**

A particular prototype function called IOTA (Isotropic Orthogonal Transform Algorithm) [13], [16] satisfying (8) is evaluated in this paper. This function is shown on Fig. 4. By construction, the IOTA function has the same shape in both time and frequency domain. To simplify the notations, we call IOTA-OFDM an OFDM/QAM system using the IOTA function. As stated before, with IOTA-OFDM real valued symbols are transmitted at twice the rate of conventional OFDM in the case of no guard interval inserted between the symbols.

**VI. Simulation Results**

In order to illustrate the performances of the PRP-OFDM approach, short range simulations have been performed comparing CP-OFDM vs. PRP-OFDM in the IEEE802.11a [1] (equivalent to HIPERLAN/2 [11]) WLAN context: a \( N = 64 \) carrier 20MHz bandwidth broadband wireless system operating in the 5.2GHz band using a 16 sample prex or postx. A rate \( R = \frac{1}{4} \), constraint length \( K = 7 \) Convolutional Code (CC) (0171/0133) is used before bit interleaving followed by QPSK constellation mapping; normalized BRAN-A [12] channel models are applied.

Fig. 5 present results where the CP-OFDM modulator has been replaced by a PRP-OFDM modulator. Each frame contains 2 known training symbols, followed by 72 OFDM data symbols. For PRP-OFDM, the postx is chosen as given by [9]. The channel estimation is performed based on PRP-OFDM postx sequences only using an averaging window over 40 OFDM symbols using QPSK constellations.

The performance results indicate that the MMSE equalization scheme leads to results that are still 0.75 dB away from the optimum performance reached with a perfect CIR knowledge. This
gap can further be reduced by increasing the averaging window. The Overlap-Add (OLA) decoding approach (low arithmetical complexity) has approx. a 1 dB penalty compared to the MMSE equalizer, but performs still better than the standard CP-OFDM case by approx. 0.2 dB at a BER of $10^{-3}$ and an averaging window of 20 OFDM symbols. ZF equalization performs poorly due to the noise amplification issue when performing carrier grid adaptation (switching from $P = 80$ carriers to $N = 64$).

For IOTA-OFDM we performed simulations in the 3GPP RAN WG1 OFDM Study Item context [14]. Channel coding is the UMTS turbo-code, constellations are QPSK for CP-OFDM and Offset-QPSK for IOTA-OFDM. To have the same spectral efficiency for both OFDM modulations, taking into account the loss due to the cyclic prefix, the turbo-code is punctured at rate 3/4 for IOTA-OFDM and rate 4/5 for OFDM. The list of parameters can be found in Table I. The frame duration is 2 ms for both modulations, this corresponds to 12 complex valued CP-OFDM symbols or 60 real valued IOTA-OFDM symbols. Over both Vehicular A and Pedestrian B channel models, IOTA-OFDM performs at least 1 dB better than CP-OFDM.

VII. CONCLUSION

As a conclusion, it can be stated that PRP-OFDM is an evolution of OFDM suitable in the context of mobility where low-complexity channel estimation with a low pilot overhead is welcome. IOTA-OFDM follows a different approach, since no prefix/postfix sequence is any more required. It therefore aims at an increase in throughput, while channel tracking still requires pilot tones. Note that most known enhancements of CP-OFDM, such as spreading techniques (MC-SS), are compatible with ZP-, PRP- and IOTA-OFDM.

<table>
<thead>
<tr>
<th>Parameters</th>
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<th>IOTA-OFDM</th>
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<tr>
<td>FFT size (points)</td>
<td>1024</td>
<td>512</td>
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<tr>
<td># of useful sub-carriers</td>
<td>705</td>
<td>301</td>
</tr>
<tr>
<td>Cyclic prefix (# samples)</td>
<td>64</td>
<td>0</td>
</tr>
<tr>
<td>OFDM sampling rate (Msamples/sec)</td>
<td>6.528</td>
<td>7.680</td>
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<tr>
<td>Bit rate</td>
<td>6.768 Ms/s</td>
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Fig. 5. BER for IEEE802.11a, BRAN channel model A, QPSK.

Fig. 6. BER for ITU Vehicular A and Pedestrian B channel models, QPSK.

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Kenan Result

Fig. 5. BER for IEEE802.11a, BRAN channel model A, QPSK.

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