A New Adjustable Hybrid Spread OFDM Modulator

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Abstract— **The improvement brought by Successive Decoding algorithms is marginal when applied to uniformly spread OFDM-CDMA systems, because of the harmonization of the Signal to Noise Ratio across the subbands at the receiver. Thus this paper proposes a new Hybrid Spread OFDM (SOFDM) transmission scheme in which the spreading of the information is adjustable and not uniform along the carrier (frequency selective). Moreover a MMSE version of the V-BLAST Successive Interference Cancellation algorithm suited for this hybrid modulator is derived. The performance of the combination of SHOFDM and MMSE V-BLAST is shown to outperform classical iterative detection algorithms for SOFDM in the realistic scenario of the 5GHz HIPERLAN/2 system.**

Keywords— **Spread OFDM, OFDM-CDMA, BLAST, HIPERLAN/2, Successive Decoding**

I. INTRODUCTION

A Multi-carrier OFDM system [1] using a Cyclic Prefix (CP) for preventing inter-block interference is known to be equivalent to multiple flat fading parallel transmission channels in the Frequency Domain (FD). In such a system, the information sent on some carriers might be subject to strong attenuations and could be unrecoverable at the receiver. This has motivated the proposal of more robust transmission schemes combining the advantages of CDMA with the strength of OFDM known as OFDM-CDMA [2], in which the information is spread across all the carriers by a precoding unitary matrix (e.g. the Walsh-Hadamard: WH transform).

This combination increases the overall frequency diversity of the modulator, so that unreliable carriers can still be recovered by taking advantage of the subbands enjoying a high Signal to Noise Ratio (SNR). Although originally proposed for a multiuser access scheme, this concept is extended to all single user OFDM systems and is referred in the sequel as Spread OFDM (SOFDM).

Due to the inter-carrier interference generated by the spreading, the frequency domain channel transfer function of a single antenna SOFDM system can be modeled using a full MIMO flat fading (scalar) matrix. Actually this is an assumption often made in multiple emitting and receiving antenna communications and already exploited in V-BLAST. Here, the advantage of OFDM systems with CP is that it validates the above assumption *even for channels with memory*.

Thus this paper presents both an extension of the Successive Interference Cancellation (SIC) algorithm V-BLAST [3] in a spirit similar to that for CDMA multiuser detection [4] and a new spreading method that combined with this new algorithm, provides an additional performance gain.

SIC algorithms relies on a sequential detection of the received block. At each step, one symbol is detected before its contribution is subtracted from the received block. This introduces successively additional degrees of freedom which enable the reduction of noise/interference influence for the next users to be detected and therefore increases the overall reliability of the decision process.

But for performing a good interference cancellation, due to the underlying feedback mechanism involved in the successive detection mechanism, one should decode first the reliable carriers enjoying a greater SNR and then the most corrupted ones. Unfortunately with a WH spreading, all the carriers share the same SNR resulting in practice to marginal performance gain when applying SIC approaches.

In order to overcome this problem, and achieve higher gains, we propose in this paper a new *adjustable* hybrid modulator scheme adopting a non uniform spreading along the carriers (frequency selective) instead of the classical uniform one, achieving a tradeoff somewhere between flat WH-OFDM and plain OFDM.

The purpose of the paper is thus twofold:

1. to propose a new adjustable flexible hybrid spreading modulator - referred in the following as SHOFDM - suited for combination with SIC techniques (section II);
2. to derive and apply to SHOFDM a new MMSE version of the original ZF V-BLAST algorithm (section III).

Section IV illustrates how the Hybrid SOFDM transmission scheme can benefit from the improved new Successive Detection (SD) MMSE-VBLAST algorithm for the ETSI BRAN HIPERLAN/2 (HL2) 5GHz local area broadband wireless system context in the uncoded scenario.

II. NEW ADJUSTABLE HYBRID SOFDM TRANSCEIVER MODEL

In the following, upper (lower boldface) symbols are used for matrices (column vectors) whereas lower symbols represent scalar values, denote by $(.)^T$ transpose operator, $(.)^\star$ conjugation and $(.)^H = \left((.)^T\right)^\star$ hermitian transpose. I_{PP} stands for the $P * P$ identity matrix.

Overall system model: Since a N carrier OFDM system [1] using a CP is equivalent in the FD to N flat fading parallel transmission channels, the baseband discrete-time block equivalent model of a SOFDM system can be depicted in figure 1. Actually the $N * 1$ received block vector $\mathbf{r} = (r_1, \cdots, r_N)^t$ can be expressed in the FD as a function of the emitted symbol $\mathbf{s} = (s_1, \cdots, s_N)^t$ and additive noise $\mathbf{b} = (b_1, \cdots, b_N)$ vectors using a MIMO flat fading channel matrix M:

$$\mathbf{r} = M\mathbf{s} + \mathbf{b} \tag{1}$$

where M consists in the product of the spreading matrix T (usually a WH transform), which can be interpreted as a source of inter-carrier interference, by the diagonal matrix $D = \mathrm{diag}(c_1, \cdots, c_N)$ of the FD channel attenuations:

$$M = DT = (\mathbf{m}_1, \cdots, \mathbf{m}_N) \tag{2}$$

A. The hybrid SOFDM scheme

Many methods for retrieving the emitted symbols have already been investigated such as conventional Zero Forcing (ZF) or Minimum Mean Square Error (MMSE) equalization, Maximum Likelihood and iterative decoding (cf.[2] and references therein).

As already mentioned in the introduction, SIC schemes do not couple very well with SOFDM because they require to be able to sort the carriers for performing decisions on the most reliable ones first. By construction, the role of a uniform spreading for SOFDM is to equalize the SNRs between the sub-bands so that no carrier can be candidate for being decoded first when using a successive decoding method. This results in practice in a poor interference cancellation coming from too important error propagation in the feedback (even when using a soft decision mechanism). This phenomenon simply annihilates the benefits of such technique.

On the other hand, in plain OFDM, it is extremely easy to find the reliable carriers due to the difference of SNR affecting the various subbands. However, in this specific case the successive decoding of the components of the received vector is of no interest since the carriers are always assumed to be independent.

These considerations inspired the proposition of a new Hybrid scheme combining the strength of SOFDM and OFDM that is suited for successive detection schemes and enhances the performance of decoding methods such as V-BLAST presented section III. The basic idea is to change the nature of the spreading and adopt an adjustable non uniform one along the carriers (frequency selective). That way, a tradeoff between flat WH-OFDM and plain OFDM is achieved. The new Hybrid modulator is therefore defined by:

$$T(\theta) = \cos(\theta)I + j\sin(\theta)W \tag{3}$$

where W denotes the Walsh-Hadamard matrix and θ is a parameter used for tuning the modulator. This adjustable new modulator deserves a few comments:

- since both I and W are unitary real matrices and $W^H W = W^2 = I$, one can verify that for all θ, $T(\theta)$ is also unitary;
- when $\theta = 0$ the overall transmitter corresponds to a classical OFDM system and when $\theta = \pi/2$ the usual Walsh-Hadamard spread OFDM system is obtained;
- any θ ($\theta \neq 0, \pi/2$) creates a new kind of diversity and can improve the successive non linear detection process;
- the above principle can be extended to any real unitary matrix verifying $W^2 = I$.

Thus, we have now at our disposal an adjustable modulator model encompassing both OFDM and WH-SOFDM. In the following a SOFDM scheme where the spreading is performed by $T(\pi/4)$ is referred as **Hybrid SOFDM: SHOFDM**.

III. MMSE Successive Interference Cancellation scheme

Taking a closer look to equation 1, one can notice that the overall SHOFDM transceiver transfer function is the matrix M. Therefore all classical detection schemes based on a MIMO flat fading channel model can be applied. Actually this is an assumption made for multiple antenna algorithms and is exploited in the V-BLAST approach [3].

Note that ZF equalization schemes for SOFDM perform poorly in presence of severe frequency selective attenuations. In that case, the ISI suppression criterion leads to large noise amplification on some carriers which is then spread by the T demodulator on all the subbands resulting in disastrous BER. MMSE detection schemes have to be considered for symbol recovery.

The goal of this section is thus both to derive a MMSE version of this ZF-based successive decoding scheme and to apply this algorithm for the decoding of SHOFDM.

The proposed algorithm relies on a sequential detection of the received block. At the first step of the method, a wiener equalization of matrix M is performed by matrix $G_1 = (M^H M + \sigma^2 I)^{-1} M^H$. Then the carrier k_1 enjoying the highest Signal to Interference plus Noise (SINR) is decoded. Assuming a perfect decision, the resulting estimated symbol $\tilde{s}_{k_1}$ is subtracted from the vector of received samples in the following manner: $\mathbf{r}_2 = \mathbf{r}_1 - \tilde{s}_{k_1}\mathbf{m}_{k_1}$. This introduces one degree of freedom for the next canceling vector choice which enables the reduction of the noise plus interference influence for increasing the decision process reliability. The complete detection algorithm can thus be summarized as follows:

initialization: (4)

$$i \leftarrow 1 \tag{5}$$

$$\mathbf{r}_1 = \mathbf{r} \tag{6}$$

$$G_1 = (M^H M + \sigma^2 I)^{-1} M^H \tag{7}$$

$$k_1 = \mathrm{argmax}_j (\mathrm{SINR}_j^{(1)}) \tag{8}$$

recursion: (9)

$$\mathbf{w}_{k_i} = \mathbf{g}_{k_i}^{(i)} \tag{10}$$

$$y_{k_i} = \mathbf{w}_{\mathbf{k_i}}\mathbf{r}_{\mathbf{i}} \tag{11}$$

$$\tilde{s}_{k_i} = Q(y_{k_i}) \tag{12}$$

$$\mathbf{r}_{\mathbf{i+1}} = \mathbf{r}_{\mathbf{i}} - \tilde{s}_{k_i}\mathbf{m}_{\mathbf{k_i}} \tag{13}$$

$$G_{i+1} = (M_i^H M_i + \sigma^2 I)^{-1} {M_i}^H \tag{14}$$

$$k_{i+1} = \mathrm{argmax}_{j \neq k_1, \cdots, k_i} (\mathrm{SINR}_j^{(i+1)}) \tag{15}$$

$$i \leftarrow i + 1 \tag{16}$$

Let define and justify now the remaining unexplained variables involved in the algorithm.

Be $L = (k_1, \cdots, k_N)$ the permutation of $(1, \cdots, N)$ specifying the order in which the components of the transmitted symbol vector $\mathbf{s}$ are extracted.

The Wiener filtering matrix G_i at iteration i is defined as:

$$G_i = \text{argmin}_W \|W^H \mathbf{r_i} - \mathbf{s_i}\|^2 = (M_{i-1}^H M_{i-1} + \sigma^2 I)^{-1} M_{i-1}^H$$

where matrix M_i denotes the matrix obtained by zeroing columns $(k_1, \cdots, k_{i-1})$ of M. Recall that $\mathbf{m_{k_i}}$ represents the k_ith column of M while $\mathbf{g}_{k_i}^{(i)}$ is the k_ith row of G_i at step i.Q represents the decision process.

The optimal selection of $\mathbf{g}_{k_i}^{(i)}$ as well as the choice of Q deserves some explanations.

It is shown in [5] that the distribution of the residual interference-plus noise (SINR) at the output of a linear MMSE filter can be considered as Gaussian. Therefore, we will assume that the output $y_j^{(i)}$ of the MMSE filter can be modeled at each step i of the algorithm by:

$$y_j^{(i)} = a_j^{(i)} s_j + n_j^{(i)}$$

where $a_j^{(i)}$ is the amplitude of the jth symbol at iteration i and $n_j^{(i)}$ is a gaussian noise of law $N(0, {\sigma_j^{(i)}}^2)$. The parameters $a_j^{(i)}$ and $\sigma_j^{(i)}$ are given by:

$$\begin{aligned} a_j^{(i)} &= \mathbf{g}_j^{(i)} \mathbf{m}_j \\ {\sigma_j^{(i)}}^2 &= \mathbf{g}_j^{(i)} {\mathbf{g}_j^{(i)}}^H \sigma^2 + \sum_{\forall p \neq k_1, \cdots, k_i}^{n} \mathbf{g}_j^{(i)} \mathbf{m}_p E(|s_p|^2) \end{aligned}$$

The SINR per symbol is thus expressed as:

$$\text{SINR}_j^{(i)} = \frac{{a_j^{(i)}}^2 E(|s_j|^2)}{{\sigma_j^{(i)}}^2}$$

Note that the chosen vector $\mathbf{g}_j^{(i)}$(equation 10) corresponds to the one leading to the highest SINR value.

Once the symbol is retrieved (equation 11), a decision is made modeled by operator Q (equation 12). Instead of performing a hard sign decision, it is often better to use for Q a “soft” one using the hyperbolic tangent non linear detector whose argument is weighted by an estimation of the SINR. Such a modification of the canceling mechanism is appealing since it tends to attenuate the effect of unreliable decisions when a low SINR occurs [4].

Finally, in the uncoded case, once all symbols have been iteratively retrieved, a hard decision is performed on the resulting vector $\mathbf{y} = (y_1, \cdots, y_N)^t$.

IV. Simulation results

This section compares the performance of the new MMSE V-BLAST Successive Detection (SD) algorithm applied both on SHOFDM and SOFDM systems to classical OFDM and SOFDM schemes.

Simulations have been performed in the HL2 system context using QPSK constellations. HL2 is a $N = 64$ carrier local area broadband wireless system operating in the 5GHz band over 20MHz (equivalent to IEEE802.11) using a 16 samples Cyclic Prefix. The BER versus Eb/No of the various detection algorithms are illustrated fig. 3, based on Monte Carlo simulations, with each trial corresponding to a different realization of the indoor 5GHz typical Rayleigh fading wireless Channel Model A specified by ETSI [6] and assuming perfect synchronization and channel knowledge.

Fig. 2 depicts the BER achieved for various values of θ for an SNR of 11dB (similar curve shapes are obtained for other SNRs). It confirms that in an uncoded context, $\theta = \pi/4$ is the optimum choice for achieving the best performance on average when combined with successive decoding schemes. Note that only minor enhancements can be expected in adjusting adaptively the angle to the channel realization.

Fig. 3 underlines the gain achieved with the new system over competing techniques in the uncoded case. When using a MMSE filter, the SHOFDM curve is between the plain OFDM (note that ZF and MMSE equalized OFDM give the same results for QPSK constellations) and Spread-OFDM. This is quite justified since a linear detector is used. On the other hand, for a BER of 10^{-4}, a gain of 4.5 dB is achieved over MMSE SOFDM when MMSE Successive Detection is used on the hybrid modulator. For the same detection technique, SHOFDM outperforms classical WH-SOFDM by a gain of 3 dB at BER 10^{-4}.

The lower bound theoretical curve corresponds to an ideal WH-spreading with no interference: i.e. a diagonal channel matrix where each coefficient is the summation of the squared frequency channel coefficients.

A derived version of the iterative soft block decision feedback equalizer proposed in [7] has also been compared to our new detection scheme. For these methods, it is known that the highest gain is achieved only after the first iteration. Therefore only one iteration has been plotted (curve Iterative Decoding: ID SOFDM of fig. 3). Still, a gain of 3dB is achieved with our new system. Anyway, for higher BER, greater gains are always achieved.

It has been shown in another paper [8] that the enhancement provided by SHOFDM is still preserved (2 dB gain over SOFDM) in the coded case (for rates $R > \frac{1}{2}$), if we take into account a customization of the usual Viterbi metrics.

V. Conclusion

In this contribution, we have proposed and applied a new MMSE successive interference cancellation based on the V-BLAST algorithm to a new Hybrid adjustable OFDM modulator. The diversity provided by the modification of the spreading nature provides means for great improvements over classical OFDM and SOFDM schemes at a cost of an increased arithmetical complexity for the decoding.

A gain of more than 4.5 dB is achieved over MMSE V-BLAST WH-SOFDM for a BER of 10^{-4} in the uncoded case. These results, simulated in a realistic environment (HIPERLAN/2), confirm that Hybrid SOFDM is a promising transmission technique over frequency selective fading wireless channels.

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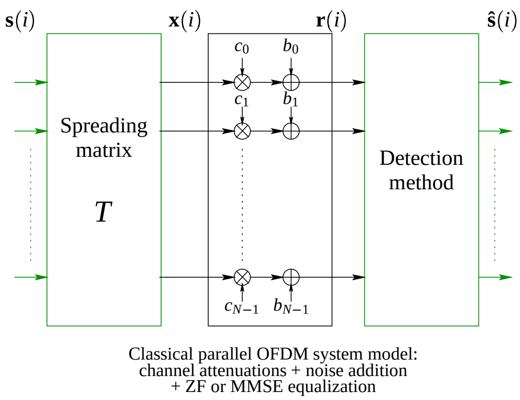

Fig. 1. Classical frequency domain SOFDM transmitter

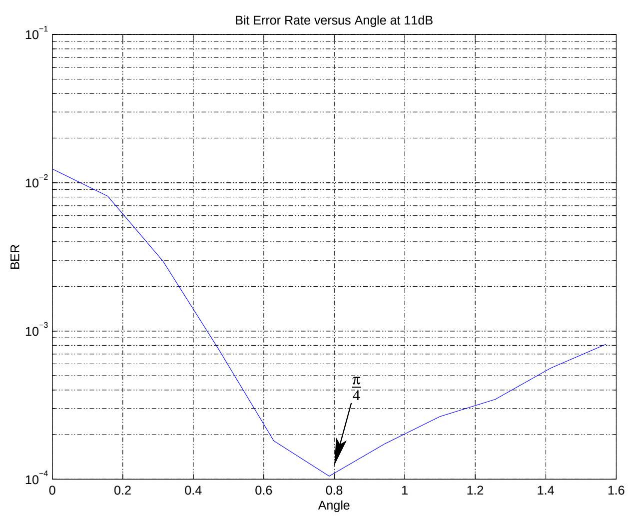

Fig. 2. BER versus θ at 11dB

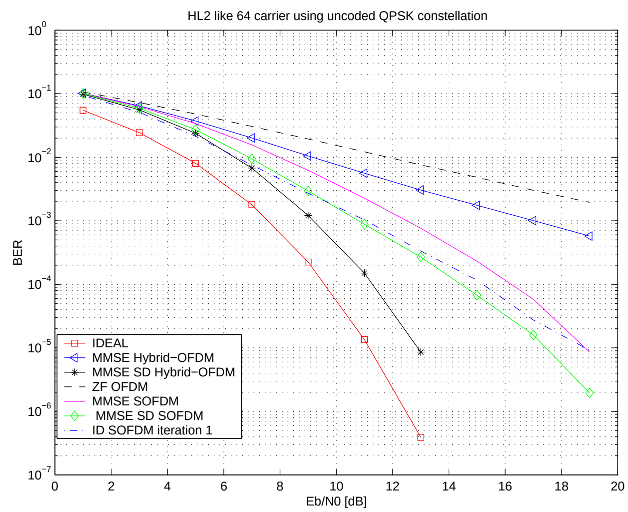

Fig. 3. Uncoded BER