Iterative Interference Suppression for Pseudo Random Postfix OFDM based Channel Estimation

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***Abstract*— This contribution[1] proposes an iterative channel impulse response (CIR) estimation scheme in the context of the Pseudo-Random-Postfix OFDM (PRP-OFDM) modulation. While conventional techniques reduce noise and interference of OFDM data symbols on the CIR estimates by simple mean value calculation, the new proposal uses soft decoder outputs in order to perform iterative OFDM data symbol interference cancellation. In a typical example for QAM-64 constellations, the mean-square-error (MSE) of the CIR estimates is improved by approx. 12dB after three iterations. Based on PRP-OFDM postfixes only, it is thus possible to perform channel estimation for higher order constellations (QAM-64 and higher) with an initial CIR estimation over a small observation window (for QAM-64 typically 30 to 40 OFDM symbols). Any loss in throughput due to additional redundancy for CIR estimation purposes, e.g. pilot tones, learning symbols, etc. is avoided at the cost of an increase in decoding complexity.**

I. Introduction

Nowadays, Orthogonal Frequency Division Multiplexing (OFDM) seems the preferred modulation scheme for modern broadband communication systems. Indeed, the OFDM inherent robustness to multi-path propagation and its appealing low complexity equalization receiver makes it suitable either for high speed modems over twisted pair (digital subscriber lines xDSL), terrestrial digital broadcasting (Digital Audio and Video Broadcasting: DAB, DVB) and 5GHz Wireless Local Area Networks (WLAN: IEEE802.11a and ETSI BRAN HIPERLAN/2) [1]–[4].

All these systems are based on a traditional Cyclic Prefix OFDM (CP-OFDM) modulation scheme. The role of the cyclic prefix is to turn the linear convolution into a set of parallel attenuations in the discrete frequency domain. Typically, coherent modulation schemes are applied which require channel estimates for the decoding in the receiver. The problem of CP-OFDM is that this estimation step usually requires learning symbols and pilot tones which reduce the system throughput. In order to avoid this drawback, a new modulation scheme has been recently proposed: the Pseudo Random Postfix OFDM (PRP-OFDM) modulation [5], [6] uses instead of a cyclic prefix extension a known vector weighted by a pseudo random scalar sequence [7]. This way, unlike for the former OFDM modulators, the receiver can exploit an additional information: the prior knowledge of a part of the transmitted block. [5] illustrates several efficient equalization and decoding schemes. [8] compares standard CP-OFDM with CIR estimation based on rotating pilot schemes to low-complexity PRP-OFDM CIR estimation: If the interference of OFDM data symbols on the CIR estimated is simply reduced by mean value calculation over a large observation window, in a typical example PRP-OFDM based CIR estimation leads to superior results in terms of mean-square-error (MSE) up to an SNR of approx. 15dB. At higher SNRs, standard PRP-OFDM based channel estimation techniques lead to unpractical requirements on the observation window size in order to meet the MSE constraints on the CIR estimates.

This paper proposes an iterative CIR estimation scheme that refines first rough CIR estimates and makes PRP-OFDM suitable for higher constellation types, i.e. QAM-64 and higher. The new scheme is complex if used for CIR estimation only, but in combination with iterative forward-error-correction (FEC) decoding schemes the system complexity only increases slightly.

The paper is organized as follows. Section II introduces the notations and defines the PRP-OFDM modulator. Channel estimation techniques are discussed in section III and section IV gives simulation results. A short conclusion follows in section V.

II. Notations and PRP-OFDM modulator

This section defines the baseband discrete-time block equivalent model of a N carrier PRP-OFDM system. The ith $N \times 1$ input digital vector [2] $\tilde{\mathbf{s}}_N(i)$ is first modulated by the IFFT matrix $\mathbf{F}_N^H = \frac{1}{\sqrt{N}}\left(W_N^{ij}\right)^H, 0 \leq i < N, 0 \leq j < N$ and $W_N = e^{-j\frac{2\pi}{N}}$. Then, a deterministic postfix vector $\mathbf{c}_D = (c_0, \ldots, c_{D-1})^T$ weighted by a pseudo random value $\alpha(i) \in \mathbb{C}, |\alpha(i)| = 1$ is appended to the IFFT outputs $\mathbf{s}_N(i)$. A pseudo random $\alpha(i)$ prevents the postfix time domain signal from being deterministic and avoids thus spectral peaks [5]. With $P = N + D$, the corresponding $P \times 1$ transmitted vector is $\mathbf{s}_P(i) = \mathbf{F}_{\mathrm{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{c}_P$, where

$$\mathbf{F}_{\mathrm{ZP}}^H = \left[\frac{\mathbf{I}_N}{\mathbf{0}_{D,N}}\right]_{P\times N} \mathbf{F}_N^H \quad \text{and} \quad \mathbf{c}_P = \left(\mathbf{0}_{1,N}\; \mathbf{c}_D^T\right)^T$$

The samples of $\mathbf{s}_P(i)$ are then sent sequentially through the channel modeled here as a Lth-order FIR $H(z) = \sum_{n=0}^{L-1} h_n z^{-n}$

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[2]Lower (upper) boldface symbols will be used for column vectors (matrices) sometimes with subscripts N, D or P emphasizing their sizes (for square matrices only); tilde will denote frequency domain quantities; argument i will be used to index blocks of symbols; ^{H} (^{T}) will denote Hermitian (Transpose).

of impulse response $(h_0, \cdots, h_{L-1})$. The OFDM system is designed such that the postfix duration exceeds the channel memory $L \leq D$.

Let $\mathbf{H}_{\text{ISI}}(P)$ and $\mathbf{H}_{\text{IBI}}(P)$ be respectively the Toeplitz inferior and superior triangular matrices of first column: $[h_0, h_1, \cdots, h_{L-1}, 0, \rightarrow, 0]^T$ and first row $[0, \rightarrow, 0, h_{L-1}, \cdots, h_1]$. As already explained in [9], the channel convolution can be modeled by $\mathbf{r}_P(i) = \mathbf{H}_{\text{ISI}}\mathbf{s}_P(i) + \mathbf{H}_{\text{IBI}}\mathbf{s}_P(i-1) + \mathbf{n}_P(i)$. $\mathbf{H}_{\text{ISI}}(P)$ and $\mathbf{H}_{\text{IBI}}(P)$ represent respectively the intra and inter block interference. Since $\mathbf{s}_P(i) = \mathbf{F}_{\text{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{c}_P$, we have:

$$\mathbf{r}_P(i) = (\mathbf{H}_{\text{ISI}} + \beta_i \mathbf{H}_{\text{IBI}})\mathbf{s}_P(i) + \mathbf{n}_P(i)$$

where $\beta_i = \frac{\alpha(i-1)}{\alpha(i)}$ and $\mathbf{n}_P(i)$ is the ith AWGN vector of element variance σ_n^2. Note that $\mathbf{H}_{\beta_i} = (\mathbf{H}_{\text{ISI}} + \beta_i \mathbf{H}_{\text{IBI}})$ is pseudo circulant: i.e. a circular matrix whose $(D-1)\times(D-1)$ upper triangular part is weighted by β_i.

The expression of the received block is thus:

$$\begin{aligned} \mathbf{r}_P(i) &= \mathbf{H}_{\beta_\text{i}} \left(\mathbf{F}_{\text{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{c}_P \right) + \mathbf{n}_P(i) \qquad (1) \\ &= \mathbf{H}_{\beta_\text{i}} \begin{pmatrix} \mathbf{F}_N^H \tilde{\mathbf{s}}_N(i) \\ \alpha(i)\mathbf{c}_D \end{pmatrix} + \mathbf{n}_P(i) \end{aligned}$$

Please note that equation (1) is quite generic and captures also the CP and ZP (Zero Padding) modulation schemes. Indeed ZP-OFDM corresponds to $\alpha(i) = 0$ and CP-OFDM is achieved for $\alpha(i) = 0$, $\beta_i = 1 \forall i$ and $\mathbf{F}_{\text{ZP}}^H$ is replaced by $\mathbf{F}_{\text{CP}}^H$, where

$$\mathbf{F}_{\text{CP}}^H = \left[\frac{\mathbf{0}_{D,N-D} \mid \mathbf{I}_D}{\mathbf{I}_N} \right]_{P \times N} \mathbf{F}_N^H.$$

With these notations, CIR estimation is discussed in the following.

III. Channel estimation

In the sequel, the standard low-complexity PRP-OFDM CIR estimation technique [5] based on interference suppression by mean value calculation is briefly recalled. Then, the proposal of an iterative scheme follows which helps to considerably improve the MSE of the CIR estimates. All derivations are detailed for the static context; the extension to mobility scenarii is straightforward when applying the techniques presented in [6], [8].

A. Standard channel estimation

Define $\mathbf{H}_{\text{CIR}}(D) := \mathbf{H}_{\text{ISI}}(D) + \mathbf{H}_{\text{IBI}}(D)$ as the $D \times D$ circulant channel matrix of first row $row_0(\mathbf{H}_{\text{CIR}}(D)) = [h_0, 0, \rightarrow, 0, h_{L-1}, \cdots, h_1]$. Note that $\mathbf{H}_{\text{ISI}}(D)$ and $\mathbf{H}_{\text{IBI}}(D)$ contain respectively the lower and upper triangular parts of $\mathbf{H}_{\text{CIR}}(D)$.

Denoting by $\mathbf{s}_N(i) := [s_0(i), \cdots, s_{N-1}(i)]^T$, splitting this vector in 2 parts: $\mathbf{s}_{N,0}(i) := [s_0(i), \cdots, s_{D-1}(i)]^T$, $\mathbf{s}_{N,1}(i) := [s_{N-D}(i), \cdots, s_{N-1}(i)]^T$, and performing the same operations for the noise vector: $\mathbf{n}_P(i) := [n_0(i), \cdots, n_{P-1}(i)]^T$, $\mathbf{n}_{D,0}(i) := [n_0(i), \cdots, n_{D-1}(i)]^T$, $\mathbf{n}_{D,1}(i) := [n_{P-D}(i), \cdots, n_{P-1}(i)]^T$, the received vector $\mathbf{r}_P(i)$ can be expressed as:

$$\mathbf{r}_P(i) = \begin{pmatrix} \mathbf{H}_{\text{ISI}}(D)\mathbf{s}_{N,0}(i) + \alpha(i-1)\mathbf{H}_{\text{IBI}}(D)\mathbf{c}_D + \mathbf{n}_{D,0} \\ \vdots \\ \mathbf{H}_{\text{IBI}}(D)\mathbf{s}_{N,1}(i) + \alpha(i)\mathbf{H}_{\text{ISI}}(D)\mathbf{c}_D + \mathbf{n}_{D,1} \end{pmatrix}. \qquad (2)$$

As usual the transmitted time domain signal $\mathbf{s}_N(i)$ is zero-mean. Thus the first D samples $\mathbf{r}_{P,0}(i)$ of $\mathbf{r}_P(i)$ and its last D samples $\mathbf{r}_{P,1}(i)$ can be exploited very easily to retreive the channel matrices relying on the deterministic nature of the postfix as follows:

$$\begin{aligned} \hat{\mathbf{r}}_{c,0} &:= \mathrm{E}\left[\frac{\mathbf{r}_{P,0}(i)}{\alpha(i-1)}\right] = \mathbf{H}_{\text{IBI}}(D)\mathbf{c}_D, \\ \hat{\mathbf{r}}_{c,1} &:= \mathrm{E}\left[\frac{\mathbf{r}_{P,1}(i)}{\alpha(i)}\right] = \mathbf{H}_{\text{ISI}}(D)\mathbf{c}_D. \end{aligned} \qquad (3)$$

Since $\mathbf{H}_{\text{ISI}}(D) + \mathbf{H}_{\text{IBI}}(D) = \mathbf{H}_{\text{CIRC}}(D)$ is circular and diagonalizable in the frequency domain $\mathbf{F}_D$ combining equations (3) and using the commutativity of the convolution yields:

$$\begin{aligned} \hat{\mathbf{r}}_c &:= \hat{\mathbf{r}}_{c,0} + \hat{\mathbf{r}}_{c,1} = \mathbf{H}_{\text{CIRC}}(D)\mathbf{c}_D \\ &= \mathbf{C}_D\mathbf{h}_D = \mathbf{F}_D^H \tilde{\mathbf{C}}_D \mathbf{F}_D \mathbf{h}_D, \end{aligned} \qquad (4)$$

where $\mathbf{C}_D$ is a $D \times D$ circulant matrix with first row $row_0(\mathbf{C}_D) := [c_0, c_{D-1}, c_{D-2}, \cdots, c_1]$ and $\tilde{\mathbf{C}}_D := \text{diag}\{\mathbf{F}_D\mathbf{c}_D\}$.

Since in practice the expectation $\mathrm{E}[\cdot]$ in (3) is approximated by a mean value calculation over a limited number Z of symbols, we can model the estimation error as noise $\tilde{\mathbf{n}}_D$.

Assuming both the received OFDM time domain data samples and $\mathbf{n}_P$ to be Gaussian of respective covariances $\sigma_s^2 \mathbf{I}_N$ and $\sigma_n^2 \mathbf{I}_P$, the covariance of $\tilde{\mathbf{n}}_D$ is $\mathbf{R}_{\tilde{\mathbf{n}}_D} = \mathrm{E}\left[\tilde{\mathbf{n}}_D \tilde{\mathbf{n}}_D^H\right] = \frac{\sigma_s^2 + \sigma_n^2}{Z}\mathbf{I}_D$. Thus, an estimate of the CIR $\hat{\mathbf{h}}_D$ can be retrieved by either a ZF or MMSE approach [10].

The iterative estimation scheme presented in the sequel requires an initial CIR estimate which is for example obtained by the previously presented technique.

B. Novel iterative channel estimation

The iterative CIR estimation is performed in several steps:

1) Initial CIR estimation: Perform an initial CIR estimation, for example as proposed in section III-A.
2) Perform FEC decoding with latest CIR estimates: Buffer the outputs of the soft-output decoder which indicate the bit-probabilities of the lth encoded bit of the constellation on carrier n of OFDM symbol i: $p_l(x_n(i))$ with $n \in [0, \cdots, N-1]$ and $l \in [0, \cdots, log_2(Q)-1]$; Q is the constellation order.
3) Interference estimation or end: If the decoding results are acceptable, stop the procedure. Otherwise: As derived in appendix I, the interference estimation from OFDM data symbol i is calculated based on the bit-probabilities $p_l(x_n(i))$ and the latest CIR estimates as given by theorem 1.1 in appendix I.
4) Interference suppression: Subtract estimated interference from received vector $\mathbf{r}_P(i)$.

5) CIR estimation: The CIR is re-estimated, e.g. as proposed in section III-A. The result is usually better compared to the previous estimate, since the interference of the OFDM data symbols on the postfix convolved by the channel is reduced.
6) Iterate: Re-start with step 2).

The iterative CIR estimation is compatible to any FEC decoder which delivers at its output bit-probabilities of encoded information bits. These are for example SOVA (Soft-Output-Viterbi-Algorithm) decoders, etc. If such a decoder is applied for the sake of CIR estimation only, the complexity increase is considerable. However, if the proposed technique is used in a system where iterative decoding is used anyway (e.g. in the context of Turbo Codes, etc.), the additional complexity is reasonable.

IV. Simulation Results

In order to illustrate the performances of our approach, simulations have been performed in the IEEE802.11a [1] (equivalent to HIPERLAN/2 [2]) WLAN context: a $N = 64$ carrier 20MHz bandwidth broadband wireless system operating in the 5.2GHz band using a 16 sample postfix. The CP-OFDM modulator is replaced by a PRP-OFDM modulator. A rate $R = 1/2$, constraint length $K = 7$ Convolutional Code (CC) (o171/o133) is used before bit interleaving followed by QAM-64 constellation mapping.

Monte carlo simulations are run and averaged over 2500 realizations of a normalized BRAN-A [11] frequency selective channel without Doppler in order to obtain BER curves.

Based on a SOVA decoder, figure 1 illustrates for a fixed carrier-over-interference (C/I) ratio of $C/I = 24dB$ that the MSE of the CIR is decreased by approx. 12dB after three iterations using the *novel* algorithm proposed in section III-B compared to the initial estimates obtained by the algorithm proposed in section III-A. This gain varies only slightly with the size of the observation window for the mean-value calculation *postfix convolved by CIR plus noise*.

The BER results (of decoded bits) for a mean-value calculation window size of 30 and 40 symbols are given by Figure 2 and Figure 3 respectively. While an error floor remains visible for a mean-value calculation over 30 symbols, the performance is very acceptable for 40 symbols. Compared to an IEEE802.11a CP-OFDM based implementation with channel estimation over two learning symbols, a negligable gain is obtained for PRP-OFDM with mean-value calculation over 30 symbols and approx. a 1dB gain for PRP-OFDM with mean-value calculation over 40 symbols when iterating.

Note that the short size of the mean-value calculation window also makes the proposed scheme applicable in high Doppler scenarii.

V. Conclusion

A novel iterative interference cancellation scheme for PRP-OFDM based systems has been proposed. In a typical example the MSE of the resulting CIR estimated is improved by approx. 12dB over three iterations. This makes PRP-OFDM modulators applicable to higher order constellations, e.g. QAM-64, etc. For the reasons given in section IV, the proposed scheme can be applied in high mobility scenarii without losing throughput nor spectral efficiency compared to CP-OFDM systems designed for a static environment, since no additional redundancy in terms of pilot tones, learning symbols, etc. is necessary.

Appendix I
Interference estimation

Theorem 1.1: Define $\bar{\mathbf{r}}_P(i) = \mathbf{r}_P(i) - \mathbf{H}_{\beta_\mathrm{i}}\alpha(i)\mathbf{c}_P$ as a vector corresponding to $\mathbf{r}_P(i)$ after subtraction of the *postfix convolved by the CIR* and $\tilde{\mathbf{r}}_N^{eq}(i) = \tilde{\mathbf{s}}_N(i) + \tilde{\mathbf{n}}_N^{eq}$ as the equalized vector $\mathbf{r}_P(i)$ of the previous decoding step. The optimum interference estimates in the mean square error sense are given by

$$\mathbf{u}_P(i) = \sum_{\bar{\tilde{\mathbf{s}}}_N \in [X_0,\cdots,X_{Q-1}]^N} p(\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N | \tilde{\mathbf{r}}_N^{eq}(i)) \mathbf{H}_{\beta_\mathrm{i}} \mathbf{F}_{\mathrm{ZP}}^H \bar{\tilde{\mathbf{s}}}_N(i) \tag{5}$$

$X_n \in \mathbb{C}$ are the constellation amplitudes with $n \in [0, \cdots, Q-1]$.

Practical aspects:

The expression $p(\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N|\tilde{\mathbf{r}}_N^{eq}(i))$ is calculated using Bayes' rule:

$$p(\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N|\tilde{\mathbf{r}}_N^{eq}(i)) = \frac{p(\tilde{\mathbf{r}}_N^{eq}(i)|\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N) p(\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N)}{p(\tilde{\mathbf{r}}_N^{eq}(i))} \tag{6}$$

$p(\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N) = \prod_{n=0}^{N-1} p(\tilde{s}_n(i) = \bar{\tilde{s}}_n)$ is obtained by exploiting the bit-probabilities $b_l(\bar{\tilde{s}}_n)$ of the soft-decoder outputs: $p(\tilde{s}_n(i) = \bar{\tilde{s}}_n) = \prod_{l=0}^{log_2(Q)-1} p(b_l(\bar{\tilde{s}}_n))$ assuming that the bits are independent. This property is usually assured by a large interleaver. $p(\tilde{\mathbf{r}}_N^{eq}(i)|\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N)$ is given by a multivariate Gaussian probability density function (PDF) with $\mathbf{R}_{\tilde{\mathbf{n}}_N^{eq},\tilde{\mathbf{n}}_N^{eq}} = \mathrm{E}[\tilde{\mathbf{n}}_N^{eq}(\tilde{\mathbf{n}}_N^{eq})^H]$:

$$p(\tilde{\mathbf{r}}_N^{eq}(i)|\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N) = \pi^{-N} det^{-1}\left\{\mathbf{R}_{\tilde{\mathbf{n}}_N^{eq},\tilde{\mathbf{n}}_N^{eq}}\right\}$$
$$exp\left\{-(\tilde{\mathbf{r}}_N^{eq}(i) - \bar{\tilde{\mathbf{s}}}_N)^H \mathbf{R}_{\tilde{\mathbf{n}}_N^{eq},\tilde{\mathbf{n}}_N^{eq}}^{-1} (\tilde{\mathbf{r}}_N^{eq}(i) - \bar{\tilde{\mathbf{s}}}_N)\right\}$$

The expression $p(\tilde{\mathbf{r}}_N^{eq}(i))$ is calculated with (6) by exploiting $\sum_{\bar{\tilde{\mathbf{s}}} \in [X_0,\cdots,X_{Q-1}]^N} p(\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N|\tilde{\mathbf{r}}_N^{eq}(i)) = 1$.

If $\mathbf{R}_{\tilde{\mathbf{n}}_N^{eq},\tilde{\mathbf{n}}_N^{eq}}$ is diagonal (or approximated by a matrix containing its diagonal elements only), (5) can be considerably simplified, since $p(\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N|\tilde{\mathbf{r}}_N^{eq}(i)) = \prod_{n=0}^{N-1} p(\tilde{s}_n = \bar{\tilde{s}}_n|\tilde{r}_n^{eq}(i))$:

$$\mathbf{u}_P(i) = \sum_{n=0}^{N-1} \sum_{\bar{\tilde{s}}_n \in [X_0,\cdots,X_{Q-1}]} p(\tilde{s}_n = \bar{\tilde{s}}_n|\tilde{r}_n^{eq}(i)) \mathbf{H}_{\beta_\mathrm{i}} \mathbf{F}_{\mathrm{ZP}}^H \bar{\tilde{\mathbf{s}}}_N^{(n)}(i)$$

with $\bar{\tilde{\mathbf{s}}}_N^{(n)} = (0, \cdots, 0, \bar{\tilde{s}}_n(i), 0, \cdots, 0)^T$ where only the nth element is non-zero.

Proof of theorem 1.1:

Assuming that $\mathbf{u}_P(i)$ is a vector used in order to reduce the interference onto the postfix convolved by the channel in $\bar{\mathbf{r}}_P(i)$, the remaining total square error is given by

$$\epsilon^2(i) = \sum_{\bar{\tilde{\mathbf{s}}}_N \in [X_0,\cdots,X_{Q-1}]^N} p(\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N | \tilde{\mathbf{r}}_N^{eq}(i))$$

$$\mathrm{E}\, tr\left\{ \left(\bar{\mathbf{r}}_P(\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N) - \mathbf{u}_P(i)\right) \left(\bar{\mathbf{r}}_P(\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N) - \mathbf{u}_P(i)\right)^H \right\}$$

where $tr\{\cdot\}$ is the trace of a matrix. The optimum $\mathbf{u}_P(i)$ is found by setting

$$\begin{aligned}\frac{\partial \epsilon^2(i)}{\partial \mathbf{u}_P^\star(i)} &= \sum_{\bar{\tilde{\mathbf{s}}}_N \in [X_0,\cdots,X_{Q-1}]^N} p(\tilde{\mathbf{s}}_N(i) = \bar{\tilde{\mathbf{s}}}_N | \tilde{\mathbf{r}}_P^{eq}(i)) \\ &\quad \left(\mathbf{u}_P(i) - \mathbf{H}_{\beta_\mathbf{i}} \mathbf{F}_{\mathrm{ZP}}^H \tilde{\mathbf{s}}_N(i)\right) \\ &= \mathbf{0}\end{aligned}$$

This leads to the expression given by theorem 1.1. Q.E.D.

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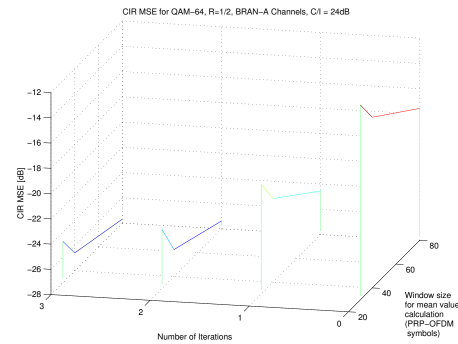

Fig. 1. CIR MSE for QAM-64, BRAN-A, C/I=24dB.

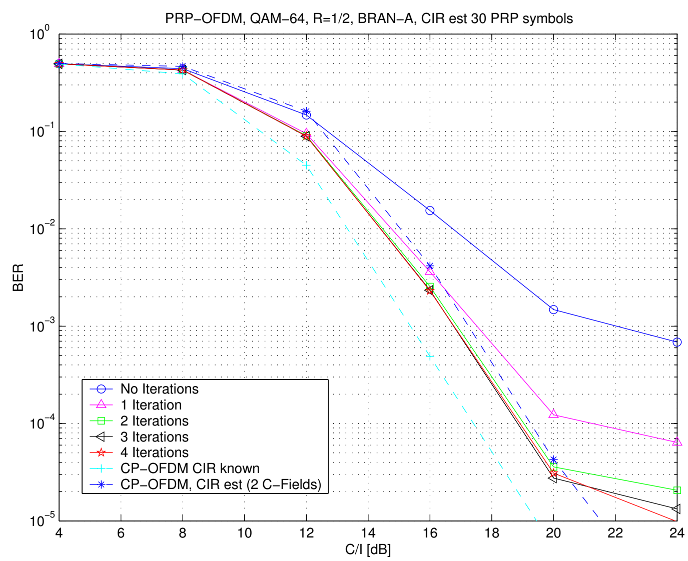

Fig. 2. Simulation results for QAM-64, R=1/2, BRAN-A, CIR estimation over 30 PRP-OFDM symbols.

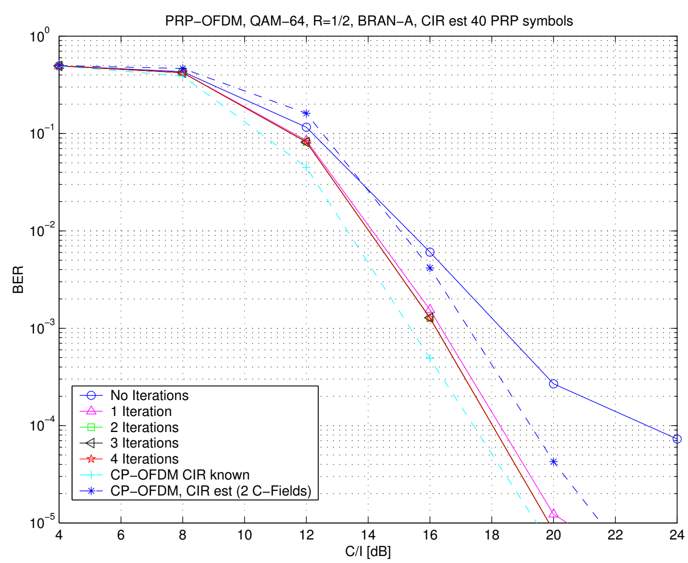

Fig. 3. Simulation results for QAM-64, R=1/2, BRAN-A, CIR estimation over 40 PRP-OFDM symbols.