Postfix Design for Pseudo Random Postfix OFDM Modulators

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***Abstract*— This contribution[1] details the derivation of suitable discrete postfix sequences for the recently proposed pseudo-random-postfix OFDM (PRP-OFDM) modulation scheme; these sequences are used in order to estimate and track the propagation channel impulse response (CIR) and have to meet the following requirements: Minimum out-of-band radiation for transmission mask requirements, spectral in-band flatness for CIR estimates with homogenous mean-square-error (MSE) over all useful carriers and low Peak-to-Average-Power-Ratio (PAPR). In order to derive such sequences, a steepest-descent based iterative optimization approach is proposed starting with the Kaiser-Window as initial sequence which is optimum except for the PAPR criterion. The proposed algorithm is applied to the 5GHz IEEE802.11a WLAN context and the final postfix sequence is evaluated by the upper criteria.**

I. Introduction

ORTHOGONAL Frequency Division Multiplexing seems the preferred modulation scheme for modern broadband communication systems. Indeed, the OFDM inherent robustness to multi-path propagation and its appealing low complexity equalization receiver makes it suitable either for high speed modems over twisted pair (digital subscriber lines xDSL), terrestrial digital broadcasting (Digital Audio and Video Broadcasting: DAB, DVB) and 5GHz Wireless Local Area Networks (WLAN: IEEE802.11a, ETSI BRAN HIPERLAN/2, IST-BROADWAY).

A main issue for any coherent OFDM system is the estimation and tracking (in a mobility scenario) of the propagation channel impulse response (CIR). Usually, this relies upon training symbols, such as preambles and pilot tones; a trade-off between the quality of the CIR estimates and the loss in throughput due to the overhead is a common issue here. Other ideas have been proposed in order to avoid this overhead, such as (semi-)blind channel tracking based on second order (or higher) statistics [2]–[4], but they usually turn out to be arithmetically complex and slowly converging. This motivated the recent proposal of the Pseudo-Random-Postfix OFDM (PRP-OFDM) modulation scheme [5], [6] as an evolution of standard Cyclic-Prefix OFDM (CP-OFDM). It offers low-complexity channel estimation and tracking means suitable to high mobility scenarii by replacing the standard cyclic prefix by a pseudo-randomly weighted sequence known to both the transmitter and the receiver. Thus, the same overhead is kept compared to CP-OFDM and the channel estimation is possible without any loss in throughput nor spectral efficiency.

In [5], [6], efficient channel estimation and decoding schemes are presented, but the actual choice of the postfix sequence remains challenging. This paper will present an efficient optimization approach which allows to derive such a sequence with respect to the following design criteria: i) minimum Peak-to-Average-Power-Ratio (PAPR), ii) minimum out-of-band radiation (important for slightly oversampled systems such as IEEE802.11a) and iii) maximum in-band flatness in order to guarantee a homogenous mean-square-error (MSE) on the frequency domain channel estimates.

This paper is organized as follows. Notations and a definition of the PRP-OFDM modulator are given section II. Section III discusses the design constraints of the postfix sequence and demonstrates that a pseudo-random-weighting of the sequence leads to preferable spectral signal properties. An iterative optimization procedure for the derivation of postfix sequences with several trade-offs (e.g. low PAPR versus in-band flatness and low out-of-band radiation) is given in section IV followed by a presentation of some examples of postfix sequences section V and simulation results in section VI. Finally, some conclusions are given section VII.

II. Notations and PRP-OFDM modulator

The baseband discrete-time block equivalent model of an N carrier PRP-OFDM system is considered as given by figure 1. The ith $N\times 1$ input digital vector[2] $\tilde{\mathbf{s}}_N(i)$ is first modulated by the IFFT matrix $\mathbf{F}_N^H := \frac{1}{\sqrt{N}}\left(W_N^{ij}\right)^H, 0\le i<N, 0\le j<N$ and $W_N := e^{-j\frac{2\pi}{N}}$.

Then, a deterministic postfix vector $\mathbf{c}_D := (c_0,\ldots,c_{D-1})^T$ weighted by a pseudo random value $\alpha(i)\in\mathbb{C}$ is appended to the IFFT output $\mathbf{s}_N(i)$. With $P := N+D$, the corresponding $P\times 1$ transmitted vector is $\mathbf{s}_P(i) := \mathbf{F}_{\mathrm{ZP}}^H\tilde{\mathbf{s}}_N(i)+\alpha(i)\mathbf{c}_P$, where

$$\mathbf{F}_{\mathrm{ZP}}^H := \left[\frac{\mathbf{I}_N}{\mathbf{0}_{D,N}}\right]_{P\times N}\mathbf{F}_N^H \quad\text{and}\quad \mathbf{c}_P := \left(\mathbf{0}_{1,N}\,\mathbf{c}_D^T\right)^T$$

Without loss of generality, the elements of $\mathbf{s}_N(i)$ are assumed to be i.i.d. and zero mean random variables of variance $\sigma_s^2=1$ which are independent of $\alpha(i)\mathbf{c}_D$. The samples of $\mathbf{s}_P(i)$ are then sent sequentially through the channel modeled here as an FIR filter of order L, $H(z) := \sum_{n=0}^{L-1} h_n z^{-n}$. The OFDM system

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[2]Lower (upper) boldface symbols will be used for column vectors (matrices) sometimes with subscripts N or P emphasizing their sizes (for square matrices only); tilde will denote frequency domain quantities; argument i will be used to index blocks of symbols; ^{H} (^{T}) will denote Hermitian (Transpose) and $(\cdot)^\star$ is the complex conjugate.

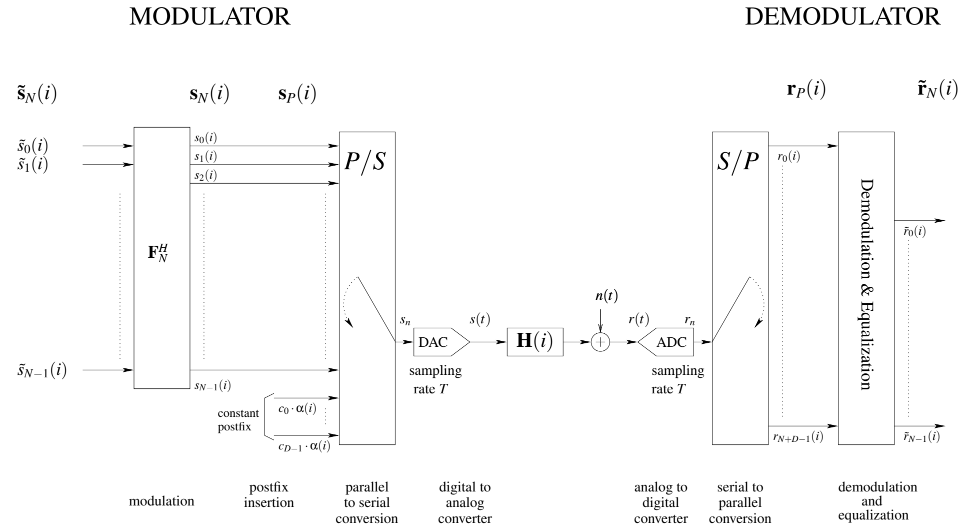

Fig. 1. Discrete model of the PRP-OFDM modulator.

is designed such that the postfix duration exceeds the channel memory $L \leq D+1$.

Let $\mathbf{H}_{\mathrm{ISI}}(P)$ and $\mathbf{H}_{\mathrm{IBI}}(P)$ be respectively the size P Toeplitz inferior and superior triangular matrices of first column $[h_0, h_1, \cdots, h_{L-1}, 0, \rightarrow, 0]^T$ and first row $[0, \rightarrow, 0, h_{L-1}, \cdots, h_1]$. As already explained in [7], the channel convolution can be modeled by $\mathbf{r}_P(i) := \mathbf{H}_{\mathrm{ISI}}(P)\mathbf{s}_P(i) + \mathbf{H}_{\mathrm{IBI}}(P)\mathbf{s}_P(i-1) + \mathbf{n}_P(i)$. $\mathbf{H}_{\mathrm{ISI}}(P)$ and $\mathbf{H}_{\mathrm{IBI}}(P)$ represent respectively the intra and inter block interference. $\mathbf{n}_P(i)$ is the ith AWGN vector of element variance σ_n^2. Since $\mathbf{s}_P(i) = \mathbf{F}_{\mathrm{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{c}_P$, we have:

$$\mathbf{r}_P(i) = (\mathbf{H}_{\mathrm{ISI}}(P) + \beta_i \mathbf{H}_{\mathrm{IBI}}(P))\mathbf{s}_P(i) + \mathbf{n}_P(i) \qquad (1)$$

where $\beta_i := \frac{\alpha(i-1)}{\alpha(i)}$. Note that $\mathbf{H}_{\beta_i} := (\mathbf{H}_{\mathrm{ISI}}(P) + \beta_i \mathbf{H}_{\mathrm{IBI}}(P))$ is pseudo circulant: i.e. a circular matrix whose $(D-1)\times(D-1)$ upper triangular part is weighted by β_i.

The expression of the received block is thus:

$$\begin{aligned} \mathbf{r}_P(i) &= \mathbf{H}_{\beta_\mathbf{i}}\left(\mathbf{F}_{\mathrm{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{c}_P\right) + \mathbf{n}_P(i) && (2) \\ &= \mathbf{H}_{\beta_\mathbf{i}} \begin{pmatrix} \mathbf{F}_N^H \tilde{\mathbf{s}}_N(i) \\ \alpha(i)\mathbf{c}_D \end{pmatrix} + \mathbf{n}_P(i) && (3) \end{aligned}$$

Considering (3), it is intuitively clear that $\mathbf{H}_{\beta_\mathbf{i}}\mathbf{c}_P$ can be retreived by a simple averaging i.e. mean value calculation of the received samples if the OFDM data symbols $\mathbf{F}_N^H \tilde{\mathbf{s}}_N(i)$ are assumed to be zero-mean. The channel can afterwards be extracted by de-convolution. This issue is discussed in detail in [5], [6], [8], [9] and will not be further presented in this paper. In the sequel, the focus is on the proper design of postfix sequences as they are assumed to be available in the given references.

III. Considerations for a proper design of the postfix

As a first postfix criterion, it is desirable that the introduction of the pseudo random postfix results in a flat spectrum of the signal sent onto the channel. In order to analyze the spectral properties of the PRP-OFDM signal since the signal is obviously not stationary but cyclostationary with periodicity P (duration of the OFDM block) [10], the order 0 cyclospectrum of the transmitted time domain sequence $s(k), k \in \mathbb{N}$ has to be calculated:

$$\mathrm{S}_{s,s}^{(0)}(z) = \sum_{k\in\mathbb{Z}} z^{-k} \frac{1}{P} \sum_{l=0}^{P-1} \mathrm{R}_{s,s}(l,k),$$

with $\mathrm{R}_{s,s}(l,k) = \mathrm{E}\left[s_{l+k}s_l^\star\right]$. Hereby, $\mathrm{R}_{s,s}(l,k)$ is given for the symbol $s(k = 0 \ldots P-1)$ as

$$\mathrm{R}_{s,s}(l,k) = \begin{cases} \mathrm{E}\left[s_{l+k}s_l^\star\right] & \text{for } k+l \geq 0 \text{ and } k+l < P \\ s_{l+k}s_l^\star \mathrm{E}_\alpha & \text{for } k+l \geq mP \text{ and} \\ & k+l < mP+D, m \in \mathbb{Z}/\{0\} \\ 0 & \text{otherwise.} \end{cases}$$

with $\mathrm{E}_\alpha = \mathrm{E}\left[\alpha\left(\lfloor\frac{l+n}{P}\rfloor\right)\alpha^\star\left(\lfloor\frac{l}{P}\rfloor\right)\right]$. Now it is clear that it is desirable to choose $\alpha(i), i \in \mathbb{Z}$ such that $\mathrm{E}_\alpha = 0$ in order to clear all influence of the deterministic postfix in the second order statistics of the transmitted signal. This is achievable by choosing $\alpha(i)$ as a pseudo-random, zero-mean value.

In order to specify the content of D samples composing the postfix we can consider the following criteria:

i) minimize the time domain peak-to-average-power ratio (PAPR);
ii) minimize out-of-band radiations, i.e. concentrate signal power on useful carriers and
iii) maximize spectral flatness over useful carriers since the channel is not known at the transmitter (do not privilege certain carriers).

The optimization of the postfix sequences from a PAPR point of view is particularly important in the case of determinstic postfix sequences: Contrarily to CP-OFDM, where PAPR related problems are of a probabilistic nature, the effects become deterministic in the PRP-OFDM context; if postfix clipping occurs in the power amplifier of the transmitter, it occurs for all postfixes. The resulting postfix is obtained through a multi-dimensional optimization involving a complex cost function. A suitable procedure is studied in detail in the following sections. Note that if the PAPR criterion is not an issue, one can directly use the Kaiser-window [11].

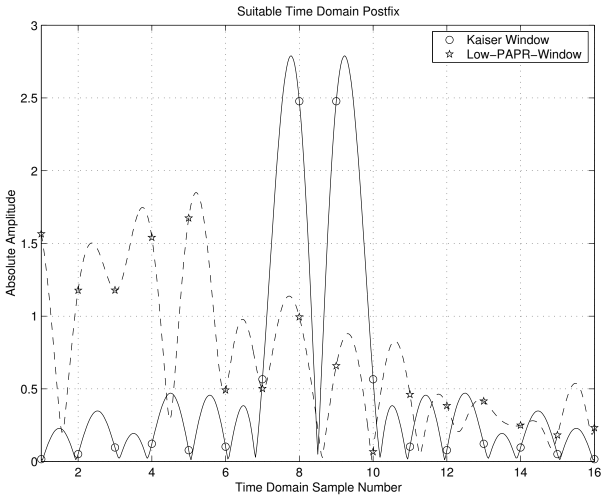

Fig. 2. Postfixes with different trade-offs in time domain.

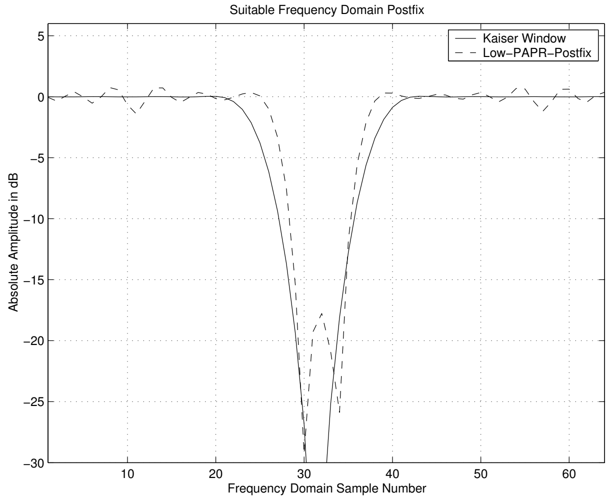

Fig. 3. Postfixes with different trade-offs in frequency domain.

IV. Iterative derivation of a suitable postfix

Since the Kaiser-Window is optimum for all criteria defined before except the PAPR criterion, the idea is to take the Kaiser-Window as initial assumption and to trade-off low PAPR against out-of-band radiation and in-band flatness by iterative steepest-descent based optimization. For this reason, a weighted cost function is defined for each criterion. In another context, such an approach is commonly applied in the field of inverse problems, e.g. by [12].

The corresponding weighted cost functions introduced are:

- $\gamma^{\text{Flat}} J^{\text{Flat}}(\mathbf{c}_D)$ with $\gamma^{\text{Flat}} \in \mathbb{R}, J^{\text{Flat}}(\mathbf{c}_D) \in \mathbb{R}$ cost function goal is to force spectral flatness over all in-band carriers;
- $\gamma^{\text{Out}} J^{\text{Out}}(\mathbf{c}_D)$ with $\gamma^{\text{Out}} \in \mathbb{R}, J^{\text{Out}}(\mathbf{c}_D) \in \mathbb{R}$ cost function aims at setting the out-of-band carriers to approximately zero;
- $\gamma^{\text{Clip}} J^{\text{Clip}}(\mathbf{c}_D)$ with $\gamma^{\text{Flat}} \in \mathbb{R}, J^{\text{Flat}}(\mathbf{c}_D) \in \mathbb{R}$ cost function role is to limit the time domain PAPR below a certain threshold.

Thus, the total cost function to be optimized is

$$J^{\text{Tot}} := \gamma^{\text{Flat}} J^{\text{Flat}}(\mathbf{c}_D) + \gamma^{\text{Out}} J^{\text{Out}}(\mathbf{c}_D) + \gamma^{\text{Clip}} J^{\text{Clip}}(\mathbf{c}_D).$$

Applying a simple steepest descent method, the minimum is found iteratively by setting $\mathbf{c}_D(i+1) = \mathbf{c}_D(i) - \nabla J^{\text{Tot}}(i)$, usually in combination with power normalization after each iteration. Hereby $\nabla J^{\text{Tot}}(i) = 2\frac{\partial}{\partial \mathbf{c}_D^\star} J^{\text{Tot}}(\mathbf{c}_D)$ with $\mathbf{c}_D = \mathbf{c}_D(i)$, where $\mathbf{c}_D$ is the vector containing the postfix of size D. The gradient of complex functions is used as defined by [13], Appendix B. In the following, both $J^{\text{Tot}}(\mathbf{c}_D)$ and $\nabla J^{\text{Tot}}(\mathbf{c}_D)$ are derived for each criterion. Since the channel is estimated over P carriers, all criteria are expressed in the $P \times P$ Fourier domain.

A. Spectral flatness

Denote by $\mathcal{C}$ the set of integers gathering the row indices of the $P \times P$ Fourier matrix $\mathbf{F}_P$ corresponding to in-band carriers and $\mathbf{F}_{\mathcal{C}}$ the submatrix of $\mathbf{F}_P$ stacking these rows. With $\mathbf{c}_P = \left(\mathbf{c}_D^T \, \mathbf{0}_{1,N}\right)^T$ and $\mathbf{f}_n^{\mathcal{C}}$ a $1 \times P$ vector containing the row of $\mathbf{F}_{\mathcal{C}}$ corresponding to carrier $\mathcal{C}_n$, i.e. the nth carrier of set $\mathcal{C}$, we have:

$$J^{\text{Flat}} := \sum_{n \in \mathcal{C}} \left[|\mathbf{f}_n^{\mathcal{C}} \mathbf{c}_P| - \frac{1}{N_{\mathcal{C}}} \sum_{k \in \mathcal{C}} |\mathbf{f}_k^{\mathcal{C}} \mathbf{c}_P| \right]^2$$

The gradient of J^{Flat} is given by $\nabla J^{\text{Flat}} = 2\left(\frac{\partial}{\partial c_0^\star}, \cdots, \frac{\partial}{\partial c_{D-1}^\star}\right)^T J^{\text{Flat}}$ with

$$\frac{\partial J^{\text{Flat}}}{\partial c_m^\star} = 2 \sum_{n \in \mathcal{C}} \left[|\mathbf{f}_n^{\mathcal{C}} \mathbf{c}_P| - \overline{c_F} \right] \left(\left(\mathbf{f}_n^{\mathcal{C}}\right)_m^\star \frac{\mathbf{f}_n^{\mathcal{C}} \mathbf{c}_P}{2|\mathbf{f}_n^{\mathcal{C}} \mathbf{c}_P|} - \partial \overline{c_F}^{(m)} \right)$$

Hereby,

$$\overline{c_F} = \frac{1}{N_{\mathcal{C}}} \sum_{n \in \mathcal{C}} |\mathbf{f}_n^{\mathcal{C}} \mathbf{c}_P|,$$

$$\partial \overline{c_F}^{(m)} = \frac{1}{N_{\mathcal{C}}} \sum_{n \in \mathcal{C}} \left(\mathbf{f}_n^{\mathcal{C}}\right)_m^\star \frac{\mathbf{f}_n^{\mathcal{C}} \mathbf{c}_P}{2|\mathbf{f}_n^{\mathcal{C}} \mathbf{c}_P|}$$

and $\left(\mathbf{f}_n^{\mathcal{C}}\right)_m^\star$ is the mth component of $\left(\mathbf{f}_n^{\mathcal{C}}\right)^\star$ and $|\mathbf{f}_n^{\mathcal{C}} \mathbf{c}_P| = \sqrt{\mathbf{f}_n^{\mathcal{C}} \mathbf{c}_P \mathbf{c}_P^H \left(\mathbf{f}_n^{\mathcal{C}}\right)^H}$.

B. Out-of-band radiation

The out-of-band radiation is defined as the power over the unused carriers and is ideally zero. With $\mathcal{O}$ being the set of $N_{\mathcal{O}} = |\mathcal{O}|$ out-of-band carriers, and $\mathbf{F}_{\mathcal{O}}$ the subset of the $\mathbf{F}_P$ Fourier matrix containing these rows $\mathbf{f}_n^{\mathcal{O}}$. The expression of the cost function is:

$$J^{\text{Out}} := \sum_{n \in \mathcal{O}} \mathbf{f}_n^{\mathcal{O}} \mathbf{c}_P \mathbf{c}_P^H \left(\mathbf{f}_n^{\mathcal{O}}\right)^H$$

The expression of J^{Out} gradient is given by $\nabla J^{\text{Out}} = 2\left(\frac{\partial}{\partial c_0^\star}, \cdots, \frac{\partial}{\partial c_{D-1}^\star}\right)^T J^{\text{Out}}$ with

$$\frac{\partial J^{\text{Out}}}{\partial c_m^\star} = \sum_{n \in \mathcal{O}} \left(\mathbf{f}_n^{\mathcal{O}}\right)_m^\star \mathbf{f}_n^{\mathcal{O}} \mathbf{c}_P.$$

C. Clipping

The impact of the clipping is determined by the transfer function of the power amplifiers (PA) in the system. In the framework of this paper, the following simple model is used:

$$f_{PA}(z) := \begin{cases} z & \text{for} \quad |z| \leq c_L \\ c_L e^{j\phi(z)} & \text{for} \quad |z| > c_L \end{cases}$$

where $c_L \in \mathbb{R}^+$ is the clipping level and $\phi(z)$ is the phase of $z \in \mathbb{C}$. The corresponding cost function is:

$$J^{\text{Clip},ideal} := \sum_{n=0}^{D-1} \left[\frac{1}{2} \left(|c_n| - c_L \right) \cdot \left[\text{sign}\left(|c_n|^2 - c_L^2 \right) + 1 \right] \right]^2 .$$

In order to further improve the resulting postfix sequence, oversampling can be applied to the postfix sequence in the upper cost function. Note that $\text{sign}\left(|c_n| - c_L\right) = \text{sign}\left(|c_n|^2 - c_L^2\right)$. For the optimization, however, we substitute $\text{sign}(x)$ by a $\mathcal{C}^1$ (differentiable) function; we choose $\text{sign}(x) \approx \tanh(\eta x), \eta \in \mathbb{R}^+$. Thus, the total cost function is

$$J^{\text{Clip}} = \sum_{n=0}^{D-1} \left[\frac{1}{2} \left(|c_n| - c_L \right) \cdot \left[\tanh\left(\eta \left(|c_n|^2 - c_L^2 \right) \right) + 1 \right] \right]^2 .$$

The gradient of J^{Clip} is given by $\nabla J^{\text{Clip}} = 2 \left(\frac{\partial}{\partial c_0^\star}, \cdots, \frac{\partial}{\partial c_{D-1}^\star} \right)^T J^{\text{Clip}}$ with

$$\begin{aligned} \frac{\partial J^{\text{Clip}}}{\partial c_m^\star} = & \frac{\left(|c_m| - c_L\right) c_m}{4|c_m|} \left[\tanh\left(\eta (|c_m|^2 - c_L^2) \right) + 1 \right]^2 + \\ & \eta \left(|c_m| - c_L \right)^2 \frac{c_m \left[\tanh\left(\eta \left(|c_m|^2 - c_L^2 \right) \right) + 1 \right]}{2\cosh^2\left(\eta (|c_m|^2 - c_L^2) \right)} \end{aligned}$$

with $m = 0, \cdots, D-1$.

Now, the total cost function is defined and a corresponding postfix can be derived by the iteration $\mathbf{c}_D(i+1) = \mathbf{c}_D(i) - \nabla J^{\text{Tot}}(i)$.

V. Example of Postfix design

Fig.2 and Fig.3 present two postfixes with different trade-offs for the parameters $D = 16$ (postfix size) and $N = 64$ (OFDM symbol size) in time and frequency domain: A Kaiser Window as given by Tab.I and a postfix whose derivation is based on the upper optimization procedure, see Tab.II. As for any IEEE802.11a based WLAN systems, the carriers $\mathcal{O} = \{28, \cdots, 38\}$ are out-of-band and $\mathcal{C} = \{1, \cdots, 27, 39, \dots 64\}$ are useful carriers. As given by Tab.III, the Kaiser Window offers optimum spectral flatness and low out-of-band radiation with the drawback of a relatively high Peak-to-Average-Power-Ratio (PAPR). The second example was derived with constraints on the PAPR; this goal is achieved, the PAPR is reduced by 3.6dB compared to the Kaiser Window at the expense of an increase in out-of-band radiation by 3.1dB and a decrease in spectral flatness: the spectral in-band ripple rises from 0.03dB (Kaiser window) to 0.92dB (low-PAPR window after optimization).

VI. Simulation Results in WLAN context

In order to illustrate the performances of our approach, simulations have been performed in the IEEE802.11a [14] (equivalent to HIPERLAN/2 [15]) WLAN context: a $N = 64$ carrier 20MHz bandwidth broadband wireless system operating in the 5.2GHz band using a 16 sample prefix or postfix. A rate $R = \frac{1}{2}$, constraint length $K = 7$ Convolutional Code (CC) (o171/o133) is used before bit interleaving followed by QPSK constellation mapping. Performance results are given for both, the classical CP-OFDM modulator and the PRP-OFDM modulator; in the CP-OFDM case, the channel estimation is performed based on two OFDM training symbols in the beginning of the frame. In the PRP-OFDM case, the CP-OFDM inherent guard interval is replaced by the above derived postfix sequence as given by Tab. II.

The decoding of the PRP-OFDM symbols in the receiver relies on the techniques presented in [5], [6], [8], [9]: The Overlap-Add (OLA) decoding approach leads to low-complexity implementation architectures, but the bit-error-rate (BER) performance remains sub-optimum; the minimum mean-square-error approach leads to an improved reliability, but the inherent algorithms are more complex. In both cases, the transformation of PRP-OFDM to ZP-OFDM is applied as presented in the given references.

Monte carlo simulations are run and averaged over 2500 realizations of a normalized BRAN-A [16] frequency selective channel without Doppler in order to obtain BER curves.

After initial acquisition, the channel estimate is then refined in the PRP-OFDM case by the semi-blind procedure explained in the paper using an averaging window over 20 and 40 OFDM symbols for QPSK constellations. Preambles or pilot tones are not used for refining the estimates. When required by the equalization structure, only a-priori knowledge of the time domain channel confinement is used concerning its statistics: $\text{E}\left[\mathbf{h}_D \mathbf{h}_D^H\right] \approx D^{-1} \mathbf{I}_D$.

For QPSK, the performance results are similar except that when doing MMSE equalization we are still 0.75 dB away from the optimum performance reached with a perfect CIR knowledge. This gap can further be reduced by increasing the averaging window. The Overlap-Add (OLA) decoding approach (low arithmetical complexity) has approx. a 1 dB penalty compared to the MMSE equalizer, but performs still better than the standard CP-OFDM case by approx. 0.2 dB at a BER of 10^{-3} and an averaging window of 20 OFDM symbols. ZF equalization performs poorly due to the noise amplification issue when performing carrier grid adaptation (switching from $P = 80$ carriers to $N = 64$).

VII. Conclusion

The design criteria for discrete postfix sequences have been discussed in the context of the Pseudo-Random-Postfix OFDM (PRP-OFDM) modulation scheme. A steepest descent based optimization algorithm has been proposed in order to trade-off different design-criteria, in particular the PAPR, the out-of-band radiation and spectral flatness. As an example, a resulting sequence is given for the IEEE802.11a WLAN context when the CP-OFDM modulator is replaced by the PRP-OFDM scheme. Simulation results show the system performance in this case exploiting only PRP-OFDM postfix sequences for channel estimation. Like this, it is possible to reduce the preamble and pilot overhead in a system design phase without any loss in system performances nor spectral efficiency.

Sample nb	Amplitude	Sample nb	Amplitude
1	0.0166 + 0.0000i	9	0.7654 - 2.3557i
2	-0.0509 + 0.0081i	10	-0.0886 + 0.5593i
3	0.0923 - 0.0300i	11	0.0000 - 0.1027i
4	-0.1089 + 0.0555i	12	-0.0123 - 0.0778i
5	0.0637 - 0.0463i	13	0.0378 + 0.1162i
6	0.0726 - 0.0726i	14	-0.0440 - 0.0864i
7	-0.3328 + 0.4581i	15	0.0303 + 0.0417i
8	1.1245 - 2.2070i	16	-0.0118 - 0.0118i

TABLE I

TIME DOMAIN SAMPLES OF A SUITABLE POSTFIX (KAISER WINDOW).

Sample nb	Amplitude	Sample nb	Amplitude
1	1.5649 - 0.0356i	9	-0.4027 - 0.5203i
2	1.1404 - 0.2923i	10	-0.0363 - 0.0561i
3	-1.1347 + 0.3148i	11	0.2141 + 0.4081i
4	1.5316 + 0.1681i	12	0.3389 + 0.1818i
5	1.6562 + 0.2440i	13	0.0789 + 0.4082i
6	0.0843 + 0.4842i	14	-0.0430 - 0.2456i
7	0.0058 - 0.5014i	15	-0.0926 - 0.1566i
8	-0.9751 - 0.1925i	16	-0.0587 - 0.2248i

TABLE II

TIME DOMAIN SAMPLES OF A SUITABLE POSTFIX (LOW PAPR).

Parameter	Kaiser Window	PAPR opt. Postfix
PAPR $\frac{\|\mathbf{c}_D\|_\infty^2}{\frac{1}{D}\sum_{n=0}^{D-1}|c_n|^2}$	9.8dB	6.2dB
Total out-of-band radiation $\frac{\sum_{n\in O}|c_n^f|^2}{\sum_{n=0}^{N-1}|c_n^f|^2}$, $\mathbf{c}^f = \mathbf{F}_N \begin{bmatrix}\mathbf{c}_D \\ \mathbf{0}_{N-D,1}\end{bmatrix}$	-17.0dB	-14.1dB
Spectral in-band ripple Calculated over carriers $\mathcal{C}^2 = \mathcal{C}\setminus\{21,\ldots 27,39,\ldots,45\}$ (i.e. transition to stop-band not considered)	0.03dB	0.92dB

TABLE III

COMPARISON ON POSTFIX TRADE-OFFS.

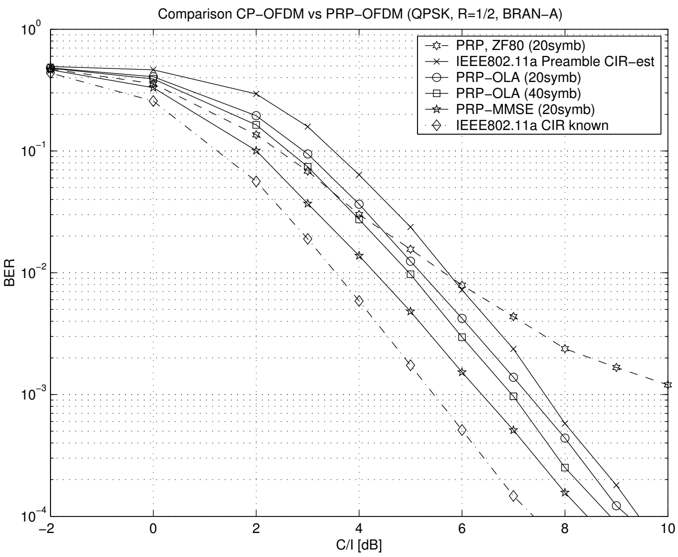

Fig. 4. BER for IEEE802.11a, BRAN channel model A, QPSK.

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