Time or Frequency Domain Pilots for Channel Tracking in Wireless OFDM Systems?

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***Abstract*— This contribution[1] compares the recently proposed Pseudo-Random-Postfix OFDM (PRP-OFDM) modulation scheme with the standard Cyclic-Prefix OFDM (CP-OFDM) modulator in the context of channel estimation and tracking. The PRP-OFDM modulation grants low-complexity channel impulse response (CIR) estimation in the receiver avoiding the usual overhead required for CP-OFDM in terms of preambles, pilot tones, etc. The evaluation criterion is the mean square error (MSE) of the CIR estimates and the overall system capacity. Results are given for the static context and for Doppler scenarios with a mobility of 30 m/s at a carrier frequency of 60 GHz.**

I. Introduction

A MAIN issue for any coherent OFDM system is the estimation and tracking (in a mobility scenario) of the propagation channel impulse response (CIR). Usually this relies upon training symbols, such as preambles and pilot tones; a trade-off between the quality of the CIR estimates and the loss in throughput due to the training symbol overhead is a common issue here. An efficient solution relies on rotating pilot schemes, as used for example in the DVB-T standard [2] combined with two-dimensional Wiener filtering (over time and frequency) for channel estimation. The corresponding algorithms are commonly known under the pseudonym *Pilot-Symbol Aided Channel Estimation (PACE)* [3],[4].

Other ideas have been proposed in order to avoid this overhead, such as (semi-)blind channel tracking based on second (or higher) order statistics [5],[6],[7], but they usually turn out to be arithmetically complex and slowly converging. This motivated the recent proposal of the Pseudo-Random-Postfix OFDM (PRP-OFDM) modulation scheme [8],[9],[10] as an evolution of standard Cyclic-Prefix OFDM (CP-OFDM) and Zero-Padded OFDM (ZP-OFDM) [11]. It offers low-complexity channel estimation and tracking means suitable for high mobility scenarios by replacing the standard cyclic prefix by a pseudo-randomly weighted deterministic sequence known to both the transmitter and the receiver. Thus the same overhead is kept compared to CP-OFDM and the channel estimation is possible without any loss in throughput.

In [10],[12] efficient and low-complexity channel estimation and decoding schemes are presented in both static and Doppler scenarios. However the important question remains which trade-offs are inherent to CIR estimation with the considered PRP-OFDM and CP-OFDM modulation schemes in terms of required overhead (learning symbols, pilots, etc.), additional power consumption and calculation complexity for given minimum mean square error (MSE) requirements on the CIR estimates. In the sequel this question will be addressed.

This paper is organized as follows. The assumed channel model is briefly introduced in Section II. Notations and definitions of CP-OFDM, ZP-OFDM, and PRP-OFDM modulators are given in Section III. In Section IV it is shown how to obtain channel observations at the receiver, which are then combined in the minimum mean square error (MMSE) sense in Section V. The considered techniques are discussed in Section VI, while some results for an example system are presented in Section VII. Section VIII concludes the paper.

II. Channel Model

We aim to model a discrete-time equivalent base-band channel. Commonly a tapped delay line model is employed, where we assume that the tap delays are multiples of $\Delta\tau$. We define a time-dependent vector[2] $\mathbf{h}(i) = [h_0(i), h_1(i), \cdots, h_m(i), \cdots h_{L-1}(i)]^T$ as the discrete-time equivalent base-band CIR. That means that the CIR has gain $h_m(i)$ at time instant $i\Delta t$ and at delay $m\Delta\tau$. In the context of OFDM we define $\Delta t = T$ and $\Delta\tau = 1/B$ to be the total OFDM symbol duration and the sampling period, respectively.

With the assumption that the arrival times of the channel taps coincide with multiples of $\Delta\tau$, the model of uncorrelated scattering can be used for the discrete-time channel, i.e. $\mathrm{E}[h_m(i)h_n^*(j)] = 0$ for $m \neq n$ and i, j arbitray. Moreover it is assumed that the tap gains follow a wide-sense stationary process with a complex Gaussian probability distribution function (PDF) with variance $\sigma_m^2 = \mathrm{E}[h_m(i)h_m^*(i)]$ and not necessarily zero mean. Hence the amplitude of each tap follows a Ricean distribution.

With the assumptions of wide-sense stationarity and uncorrelated scattering (WSS-US) we are now ready to formulate the two-dimensional auto-correlation function of the channel in a convenient fashion

$$\begin{aligned} s(m_1, m_2, i_1, i_2) &= \mathrm{E}[h_{m_1}(i_1)\, h_{m_2}^*(i_2)] \\ &= \mathrm{E}[h_{m_1}(i_1)\, h_{m_1}^*(i_2)] \cdot \delta(m_2 - m_1) \\ &= r(i_1, i_2)\delta(m_2 - m_1)\sigma_{m_1}^2 \end{aligned} \tag{1}$$

[1]This work is supported by the European Commission and part of the IST BroadWay project IST-2001-32686 [1].

[2]Lower (upper) boldface symbols will be used for column vectors (matrices) sometimes with subscripts N, P, or D emphazising their sizes; $\tilde{}$ will denote frequency domain quantities; ^{H} (^{T}) will denote Hermitian (Transpose) and $(\cdot)^*$ is the complex conjugate; $\mathbf{I}_N$ ($\mathbf{0}_{N\times M}$) stands for the size N identity (size $N \times M$ zero) matrix.

with $\delta(.)$ being the Kronecker Delta-function. The auto-correlation function in time direction $r(i_1, i_2)$ is usually selected according to Jakes' model, hence $r(i_1, i_2) = J_0(2\pi f_D(i_2 - i_1)\Delta t)$ with f_D being the Doppler frequency and J_0 being the Bessel function of first kind and zeroth order. Since the two-dimensional auto-correlation function as well as the auto-correlation function in time direction only depends on the time difference $(i_2 - i_1)\Delta t$, they are in the following expressed by $s(m_1, m_2, i_2 - i_1)$ and $r(i_2 - i_1)$, respectively.

Let $\mathbf{h}_N(i)$ be a vector composed of $\mathbf{h}(i)$ and $(N - L)$ trailing zeros. Further let $\mathbf{F}_N$ be the N-point FFT matrix with element (i, j) being $\sqrt{N}\xi^{-ij}$, where ξ is a Nth root of unity, i.e. $\xi = \exp(j2\pi/N)$. Then $\tilde{\mathbf{h}}_N(i) = \mathbf{F}_N\,\mathbf{h}_N(i) = [\tilde{h}_0(i), \tilde{h}_1(i), \cdots, \tilde{h}_{N-1}(i)]^T$ comprises the channel coefficients in the N-point frequency grid. It follows that $\tilde{h}_k(i) = \sqrt{N}\sum_{m=0}^{L-1} h_m(i)\xi^{-mk}$. It is then straightforward to obtain the two-dimensional auto-correlation function of the frequency domain channel coefficients

$$\begin{aligned} S(k_2 - k_1, i_2 - i_1) &= \mathrm{E}[\tilde{h}_{k_1}(i_1)\tilde{h}^*_{k_2}(i_2)] \\ &= \sqrt{N}\sum_{m=0}^{L-1} s(m, m, i_2 - i_1)\xi^{m(k_2 - k_1)}. \end{aligned} \tag{2}$$

III. OFDM Concepts

The baseband discrete-time block equivalent model of an OFDM system with N sub-carriers is considered. The ith $N \times 1$ input vector $\tilde{\mathbf{x}}(i)$ is first modulated by the IFFT matrix $\mathbf{F}_N^H$ and then multiplied with a $P \times N$ size guard inserting matrix $\mathbf{T}$ in order to prevent inter-symbol interference among the OFDM symbols. With $P = N + D$ the corresponding $P \times 1$ size time domain vector of the ith OFDM symbol is

$$\mathbf{x}(i) = \mathbf{T}\mathbf{F}_N^H\tilde{\mathbf{x}}(i). \tag{3}$$

In case of standard CP-OFDM the transmitted signal is preceded by a cyclic extension of D samples, thus $\mathbf{T}$ is

$$\mathbf{T}^{\mathrm{CP}} := \begin{bmatrix} \mathbf{0}_{D\times N-D}|\mathbf{I}_D \\ \mathbf{I}_N \end{bmatrix}.$$

The use of the cyclic extension simplifies the equalization process considerably, but it leads to strong sensitivity to channel nulls [11]. For the sake of robustness ZP-OFDM was proposed [6], whereby symbol recovery is always possible regardless of any channel nulls [7]. In case of ZP-OFDM the transmitted signal is followed by a zero field, hence $\mathbf{T}$ is composed of

$$\mathbf{T}^{\mathrm{ZP}} := \begin{bmatrix} \mathbf{I}_N \\ \mathbf{0}_{D\times N} \end{bmatrix}.$$

In [8],[9],[10] the PRP-OFDM concept was proposed, where the trailing zeros in ZP-OFDM are replaced by a constant vector weighted by a pseudo random sequence $\{\alpha(i)\}$, $\alpha(i) \in \mathbb{C}$, $|\alpha(i)| = 1$. The transmitted signal in a PRP-OFDM system is then obtained by adding the pseudo randomly weighted postfix to the transmitted vector in a ZP-OFDM system

$$\mathbf{x}(i) = \mathbf{T}^{\mathrm{ZP}}\mathbf{F}_N^H\tilde{\mathbf{x}}(i) + \alpha(i)\mathbf{c}_P \qquad \mathbf{c}_P := \left(\mathbf{0}_{1\times N} \mid \mathbf{c}_D^T\right)^T \tag{4}$$

with $\mathbf{c}_D$ containing a fixed sequence of length D.

As already explained in [13], the channel including the convolution with the CIR can be modeled by

$$\mathbf{y}(i) = \mathbf{H}_P^{\mathrm{ISI}}(i)\,\mathbf{x}(i) + \mathbf{H}_P^{\mathrm{IBI}}(i)\,\mathbf{x}(i-1) + \mathbf{v}_P(i), \tag{5}$$

where $\mathbf{H}_P^{\mathrm{ISI}}(i)$ and $\mathbf{H}_P^{\mathrm{IBI}}(i)$ are the size P Toeplitz inferior and superior triangular matrices of first column $[h_0(i), h_1(i), \cdots, h_{L-1}(i), 0, \cdots, 0]^T$ and first row $[0, \cdots, 0, h_{L-1}(i), \cdots, h_1(i)]$, respectively. They represent the intra and inter block interference, respectively. The vector $\mathbf{v}_P(i)$ introduces additive white Gaussian noise (AWGN), thus its elements follow a Gaussian PDF with zero mean and variance σ_v^2.

Once the CIR is known at the receiver PRP-OFDM can be reduced to ZP-OFDM [9], which in turn can be transformed to CP-OFDM by an overlap-add (OLA) approach [11]. Consequently a low-cost trade-off of PRP-OFDM is rather comparable with CP-OFDM in terms of complexity and performance, but addional information can be exploited through the prior knowledge of the postfix. Of course other performance/complexity trade-offs are possible, e.g. all known ZP-OFDM equalizers [7],[11] as well as direct equalization [9] might be applied in a PRP-OFDM system.

IV. Channel Observations at the Receiver

A. CP-OFDM

At the receiver the cyclic prefix of the received signal is discarded, thus for $L \leq D$ the remaining $N \times 1$ size vector becomes [11]

$$\begin{aligned} \mathbf{y}(i) &= \mathbf{H}_N^{\mathrm{ISI}}(i)\mathbf{F}_N^H\,\tilde{\mathbf{x}}(i) + \mathbf{H}_N^{\mathrm{IBI}}(i)\mathbf{F}_N^H\,\tilde{\mathbf{x}}(i) + \mathbf{v}_N(i) \\ &= \mathbf{H}_N^{\mathrm{CIRC}}(i)\mathbf{F}_N^H\tilde{\mathbf{x}}(i) + \mathbf{v}_N(i), \end{aligned} \tag{6}$$

where $\mathbf{H}_N^{\mathrm{CIRC}}(i) = \mathbf{H}_N^{\mathrm{ISI}}(i) + \mathbf{H}_N^{\mathrm{IBI}}(i)$ is a circular matrix that can be diagonalized by pre- and post-multiplication with FFT and IFFT matrices [14]. Hence in frequency domain we have

$$\begin{aligned} \tilde{\mathbf{y}}(i) &= \mathbf{F}_N\mathbf{H}_N^{\mathrm{CIRC}}(i)\mathbf{F}_N^H\tilde{\mathbf{x}}(i) + \mathbf{F}_N\mathbf{v}_N(i) \\ &= \mathrm{diag}\{\tilde{h}_0(i)\cdots\tilde{h}_k(i)\cdots\tilde{h}_{N-1}(i)\}\tilde{\mathbf{x}}_N(i) + \tilde{\mathbf{v}}_N(i). \end{aligned} \tag{7}$$

From (7) it follows that the channel coefficient $\tilde{h}_k(i)$ can be directly observed if the transmitted symbol $\tilde{x}_k(i)$ is known at the receiver. We assume a constant-energy-type modulation of $\tilde{x}_k(i)$, i.e. $\tilde{x}_k(i) = e^{j\varphi_k(i)}$. After back-rotating the phase on the considered sub-carrier one obtains an observation of the channel coefficient

$$\bar{\tilde{h}}_k(i) = \tilde{h}_k(i) + e^{-j\varphi_k(i)}\tilde{v}_k(i). \tag{8}$$

The phase rotation of the noise term does not change its statistical properties. Thus the observations are superimposed by samples of a white Gaussian noise process with zero mean and variance σ_v^2.

B. PRP-OFDM

We combine (4) and (5) and remark that $\mathbf{H}_P^{\mathrm{IBI}}\mathbf{T}^{\mathrm{ZP}} = \mathbf{0}_{P\times N}$ for $D \geq L$. Then the received $P \times 1$ size PRP-OFDM signal vector becomes

$$\begin{aligned} \mathbf{y}(i) &= \mathbf{H}_P^{\mathrm{ISI}}(i)\,\mathbf{T}^{\mathrm{ZP}}\mathbf{F}_N^H\,\tilde{\mathbf{x}}(i) \\ &\quad + \mathbf{H}_P^{\mathrm{ISI}}(i)\,\alpha(i)\mathbf{c}_P + \mathbf{H}_P^{\mathrm{IBI}}(i)\,\alpha(i-1)\mathbf{c}_P + \mathbf{v}_P(i). \end{aligned} \tag{9}$$

Now let $\mathbf{x}_0(i)$ and $\mathbf{x}_1(i)$ be two vectors containing the first and the last D samples of $\mathbf{F}_N^H\,\tilde{\mathbf{x}}(i)$, respectively. Employing the central limit theorem the PDF of the elements of $\mathbf{F}_N^H\,\tilde{\mathbf{x}}(i)$ (and thus also of $\mathbf{x}_0(i)$ and $\mathbf{x}_1(i)$) can be approximated by a Gaussian PDF with zero mean and variance σ_x^2. Further let $\mathbf{y}_0(i)$ and $\mathbf{y}_1(i)$ be two vectors containing the first and the last D samples of $\mathbf{y}(i)$, respectively. Similarly let $\mathbf{v}_0(i)$ and $\mathbf{v}_1(i)$ comprise the first and the last D samples of $\mathbf{v}_P(i)$, respectively. Then from (9) it follows

$$\begin{aligned}\mathbf{y}_0(i) &= \mathbf{H}_D^{\mathrm{ISI}}(i)\,\mathbf{x}_0(i) + \mathbf{H}_D^{\mathrm{IBI}}(i)\,\alpha(i-1)\mathbf{c}_D + \mathbf{v}_0(i)\\ \mathbf{y}_1(i) &= \mathbf{H}_D^{\mathrm{ISI}}(i)\,\alpha(i)\mathbf{c}_D + \mathbf{H}_D^{\mathrm{IBI}}(i)\,\mathbf{x}_1(i) + \mathbf{v}_1(i).\end{aligned} \tag{10}$$

An observation of the CIR now might be obtained by

$$\begin{aligned}\bar{\mathbf{h}}_D(i) &= \mathbf{y}_0(i)/\alpha(i-1) + \mathbf{y}_1(i)/\alpha(i)\\ &= \mathbf{H}_D^{\mathrm{ISI}}(i)\mathbf{c}_D + \mathbf{H}_D^{\mathrm{IBI}}(i)\mathbf{c}_D + \mathbf{u}_D(i)\end{aligned} \tag{11}$$

with

$$\mathbf{u}_D(i) = \frac{\mathbf{H}_D^{\mathrm{ISI}}(i)\,\mathbf{x}_0(i) + \mathbf{v}_0(i)}{\alpha(i-1)} + \frac{\mathbf{H}_D^{\mathrm{IBI}}(i)\,\mathbf{x}_1(i) + \mathbf{v}_1(i)}{\alpha(i)} \tag{12}$$

being a $D\times 1$ complex noise vector with element variance $\sigma_u^2 = \sigma_x^2 + 2\sigma_v^2$. The sum $\mathbf{H}_D^{\mathrm{ISI}}(i) + \mathbf{H}_D^{\mathrm{IBI}}(i)$ yields a circular matrix $\mathbf{H}_D^{\mathrm{CIRC}}(i)$. Hence the expression in (11) turns out to be a circular convolution of the postfix with the CIR. Using the commutative property of the convolution we obtain

$$\bar{\mathbf{h}}_D(i) = \mathbf{C}_D^{\mathrm{CIRC}}\mathbf{h}_D(i) + \mathbf{u}(i), \tag{13}$$

with $\mathbf{C}_D^{\mathrm{CIRC}}$ being a circular matrix with first row $[c_0, c_1, \cdots, c_{D-1}]$.

Contrarily to CP-OFDM with PACE an instant observation of the whole channel is obtained, though the observations are superimposed by additional noise. Hence in static or slow time-varying environments sufficiently good channel estimates might be obtained by simply averaging over a number of OFDM symbols and deconvolving with the postfix [10].

Finally it is necessary to transform the estimated CIR into the frequency domain. By limiting the CIR to the first D samples the noise term is reduced by the factor D/N [10].

It is also possible to directly observe the frequency domain channel coefficients [10]. This way a proper postfix design [15] allows a more efficient exploitation of the pilots, but complexity is usually increased.

V. MMSE Channel Estimation and Estimation Error

In the previous section it was shown how to observe the channel at the receiver. Those observations do not always directly give all channel coefficient estimates with sufficiently high accuracy required for equalization. Hence different observations have to be combined. In this article we only focus on the estimator being optimum in the MMSE sense, which is known as the two-dimensional Wiener filter [4].

A. Wiener Filtering

We briefly summarize the design of a Wiener filter and give an expression for the resulting MSE (for details see [16] and [4]). The Wiener filter (or optimal filter) processes an observation vector $\bar{\mathbf{x}}$ to obtain a vector $\hat{\mathbf{x}}$, which is an estimation of $\mathbf{x}$. The processing system is usually constrained to FIR filters. Hence all observations are combined linearly

$$\hat{\mathbf{x}} = \mathbf{W}\bar{\mathbf{x}} \tag{14}$$

with $\mathbf{W}$ being the coefficient matrix obtained by

$$\mathbf{W} = \mathbf{R}_{\mathbf{x}\bar{\mathbf{x}}}\mathbf{R}_{\bar{\mathbf{x}}\bar{\mathbf{x}}}^{-1}. \tag{15}$$

The matrixes $\mathbf{R}_{\bar{\mathbf{x}}\bar{\mathbf{x}}} = \mathrm{E}[\bar{\mathbf{x}}\bar{\mathbf{x}}^H]$ and $\mathbf{R}_{\mathbf{x}\bar{\mathbf{x}}} = \mathrm{E}[\mathbf{x}\bar{\mathbf{x}}^H]$ are respectively the auto-covariance matrix of the input vector and the cross-covariance matrix of the desired output and the input vector. The resulting (minimized) MSE of the output is given by [16]

$$\mathrm{MSE} = \frac{||\mathbf{x}-\hat{\mathbf{x}}||^2}{||\mathbf{x}||^2} = \frac{\mathrm{tr}\left(\mathbf{R}_{\mathbf{x}\mathbf{x}} - \mathbf{R}_{\mathbf{x}\bar{\mathbf{x}}}\mathbf{R}_{\bar{\mathbf{x}}\bar{\mathbf{x}}}^{-1}\mathbf{R}_{\mathbf{x}\bar{\mathbf{x}}}^H\right)}{\mathrm{tr}\left(\mathbf{R}_{\mathbf{x}\mathbf{x}}\right)} \tag{16}$$

with $\mathbf{R}_{\mathbf{x}\mathbf{x}} = \mathrm{E}[\mathbf{x}\mathbf{x}^H]$ being the auto-covariance matrix of the desired output vector and $\mathrm{tr}(.)$ being the usual trace-function.

B. Channel Estimation in Frequency Domain

Direct observations of the frequency domain channel coefficients are easily obtained by (8) for CP-OFDM and might also be obtained for PRP-OFDM [8]. Let $\boldsymbol{i} = [i_0, i_1, \cdots, i_{M-1}]$ and $\tilde{\boldsymbol{i}} = [\tilde{i}_0, \tilde{i}_1, \cdots, \tilde{i}_{M-1}]$ be two vectors containing respectively the time and frequency indices of M observed channel coefficients. Analogously let $\boldsymbol{j} = [j_0, j_1, \cdots, j_{K-1}]$ and $\tilde{\boldsymbol{j}} = [\tilde{j}_0, \tilde{j}_1, \cdots, \tilde{j}_{K-1}]$ be two vectors containing respectively the time and frequency indices of the K channel coefficients to be estimated. The above notation allows us to form an observation vector $\bar{\tilde{\mathbf{g}}} = [\bar{\tilde{h}}_{\tilde{i}_0}(i_0), \bar{\tilde{h}}_{\tilde{i}_1}(i_1), \cdots, \bar{\tilde{h}}_{\tilde{i}_{M-1}}(i_{M-1})]^T$ as well as a vector comprising the desired channel coefficient estimates $\tilde{\mathbf{g}} = [\tilde{h}_{\tilde{j}_0}(j_0), \tilde{h}_{\tilde{j}_1}(j_1), \cdots, \tilde{h}_{\tilde{j}_{K-1}}(j_{K-1})]^T$. According to the previous subsection we need to calculate the covariance matrices $\mathbf{R}_{\bar{\tilde{\mathbf{g}}}\bar{\tilde{\mathbf{g}}}}$ and $\mathbf{R}_{\tilde{\mathbf{g}}\bar{\tilde{\mathbf{g}}}}$. Their elements are easily calculated by utilizing the channel correlation functions derived in Section II

$$R_{\bar{\tilde{\mathbf{g}}}\bar{\tilde{\mathbf{g}}}}(k,m) = S(\tilde{i}_k - \tilde{i}_m, i_k - i_m) + \sigma_n^2\,\delta(m-k) \tag{17}$$

$$R_{\tilde{\mathbf{g}}\bar{\tilde{\mathbf{g}}}}(k,m) = S(\tilde{i}_m - \tilde{j}_k, i_m - j_k) \tag{18}$$

with σ_n^2 equal to σ_v^2 in the CP-OFDM case. The filter coefficient matrix can now be computed by (15) and the MSE of the estimates is obtained by (16).

C. Channel Estimation in Time Domain

Unlike in the previous subsection no direct channel observations are available, but the CIR circularily convolved with the postfix is observed.

We aim to estimate the CIR $\mathbf{h}(j)$. Let $\boldsymbol{i} = [i_0, i_1, \cdots i_{M-1}]$ be a vector comprising the time indices where observations of the entire CIR are available and form the observation vector $\bar{\mathbf{g}} = [\bar{\mathbf{h}}(i_0)^T, \bar{\mathbf{h}}(i_1)^T, \cdots, \bar{\mathbf{h}}(i_{M-1})^T]^T$. A simple argument shows that the kth element of $\bar{\mathbf{g}}$ is $\bar{h}_{k \bmod D}(k \operatorname{div} D)$ with mod and div yielding the remainder and the integer part of a division, respectively. Analogously we define $\mathbf{g} = [\mathbf{h}(i_0)^T, \mathbf{h}(i_1)^T, \cdots, \mathbf{h}(i_{M-1})^T]^T$ and $\mathbf{w} = [\mathbf{u}(i_0)^T, \mathbf{u}(i_1)^T, \cdots, \mathbf{u}(i_{M-1})^T]^T$. Considering (13) it is easy to show that $\bar{\mathbf{g}} = (\mathbf{I}_M \otimes \mathbf{C}_D^{\mathrm{CIRC}})\mathbf{g} + \mathbf{w}$ with $\otimes$ being the

Kronecker product. With the above notations we can express the covariance matrices $\mathbf{R}_{\bar{\mathbf{g}}\bar{\mathbf{g}}}$ and $\mathbf{R}_{\mathbf{h}\bar{\mathbf{g}}}$ as

$$\mathbf{R}_{\bar{\mathbf{g}}\bar{\mathbf{g}}} = (\mathbf{I}_M \otimes \mathbf{C}_D^{\mathrm{CIRC}})\mathbf{R}_{\mathbf{gg}}(\mathbf{I}_M \otimes \mathbf{C}_D^{\mathrm{CIRC}})^H + \mathbf{R}_{\mathbf{ww}} \quad (19)$$

$$\mathbf{R}_{\mathbf{h}\bar{\mathbf{g}}} = \mathbf{R}_{\mathbf{hg}}(\mathbf{I}_M \otimes \mathbf{C}_D^{\mathrm{CIRC}})^H, \quad (20)$$

where $\mathbf{R}_{\mathbf{ww}} = \sigma_u^2 \mathbf{I}_{M \cdot D}$ and the elements of $\mathbf{R}_{\mathbf{gg}}$ and $\mathbf{R}_{\mathbf{gh}}$ are calculated by

$$R_{\mathbf{gg}}(k,m) = s(k \bmod D, m \bmod D, (k-m) \operatorname{div} D) \quad (21)$$

$$R_{\mathbf{hg}}(k,m) = s(k, m \bmod D, (m \operatorname{div} D) - j). \quad (22)$$

Finally the filter coefficient matrix $\mathbf{W}$ and the MSE of the estimates are obtained by (15) and (16), respectively.

VI. Discussion

Recent contributions [8],[9] demonstrated quite impressively the performance capabilities of the semi-blind channel estimation technique based on PRP-OFDM. The clear advantage of PRP-OFDM is that channel estimation is possible without reducing the throughput, i.e. no additional overhead in terms of bandwidth is required. Moreover at the receiver D pilot samples can be exploited, which usually is considerably more than in a CP-OFDM system employing PACE. This in turn means that usually in PRP-OFDM more power has to be spent for the pilot symbols than in CP-OFDM. We conclude that PRP-OFDM is bandwidth efficient and CP-OFDM in conjunction with PACE is power efficient.

We next try to draw a more general conclusion about the efficiency of both techniques by considering the different effective bandwidth and power consumption. For this reason the total system capacity is calculated.

Recall that CP-OFDM and ZP-OFDM are quasi-equivalent in terms of performance/complexity if OLA is employed in a ZP-OFDM receiver [11]. Say in either case N_D sub-carriers are used to transmit information. In addition N_P pilots are transmitted to estimate the channel. These N_P pilots are either reserved sub-carriers in a CP-OFDM system or pilot samples in the guard interval in a ZP-OFDM system. From the total power consumption point of view that means that a certain portion of the transmitted power is spent for the pilot symbols. Thus the remaining power for the data symbols is reduced by the factor $N_D/(N_D + N_P)$. Moreover imperfect channel estimation introduces an additional noise term with its variance quantified by the MSE. Hence the total effective signal-to-noise ratio (SNR) on the data sub-carriers amounts to

$$\mathrm{SNR}_{\mathrm{eff}} = \left(\left(\mathrm{SNR} \frac{N_D}{N_D + N_P} \right)^{-1} + \mathrm{MSE} \right)^{-1}. \quad (23)$$

with SNR being the usual channel SNR. We now utilize Shannon's well known capacity formula for Gaussian channels

$$C = \frac{N_D}{N} B\,T \ \log_2(1 + \mathrm{SNR}_{\mathrm{eff}}) \text{ [bit/OFDM symbol]} \quad (24)$$

with B and T being the total system bandwidth and the symbol duration, respectively. The factor N_D/N considers that only a part of the total bandwidth is used to transmit information.

As a result of this section we established a measurement allowing a fair comparison of different channel estimation techniques considering their estimation accuracy, additional power consumption, and effective bandwidth for data transmission.

VII. Performance Evaluation

In this section we compare the performance of the two channel estimation techniques. An example OFDM system similar to HiPerLAN/2 [17] and IEEE802.11a [18] but operating at 60 GHz is considered [1]. The system bandwidth is 20 MHz and 52 out of 64 sub-carriers are used. Since at 60 GHz the channels become quite short, the guard interval is fixed to 400 ns (8 samples). Hence the total duration of one OFDM symbol is 3.6 μs.

In case of CP-OFDM with PACE we use the elements of the following set as pilot positions: $\{\pm 2, \pm 6, \pm 10, \pm 14, \pm 18, \pm 22, \pm 24, \pm 26\}$ and distribute the pilots equally over a certain number of OFDM symbols. Three different scenarios will be considered: 2, 4, and 8 pilots per OFDM symbol.

For PRP-OFDM we use the Kaiser window of length 8 as the postfix [15]. As mentioned in Section IV-B the transformation of the final CIR estimate into the frequency domain involves a reduction of the MSE by the factor D/N. In our example system this amounts to 9 dB.

The Wiener filter requires the knowledge of the power delay profile (PDP) of the CIR. We assume a CIR of length D with a rectangular PDP. Moreover it is assumed that the Doppler frequency as well as the SNR is perfectly known at the receiver. For the estimation of all required channel coefficients at one time instant the observations of 20 OFDM symbols are considered and a delay of 2 OFDM symbols is adopted. It shall be noted that the complexity of the two schemes is of the same order and directly depends on the number of pilots per OFDM symbol.

The MSE of the channel coefficient estimates is calculated using (16). Simulations confirm the theoretical results, but they are ommited in this paper due to limited space.

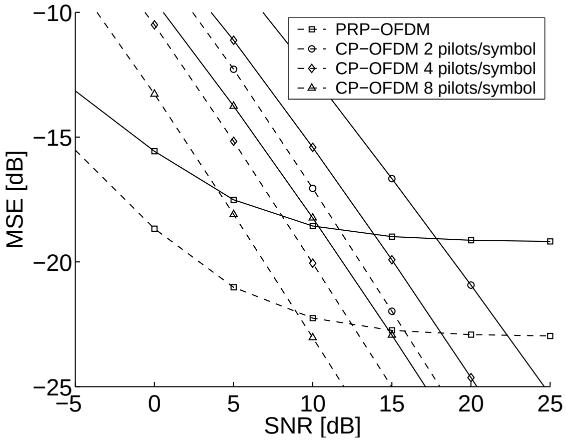

Fig. 1. MSE as a function of the SNR (dashed lines: v=0, solid lines v=30m/s)

We now focus on the achievable MSE of the channel estimates. In Fig. 1 the MSE is plotted as a function of the SNR for a static environment and for a Doppler environment with 30 m/s relative speed between transmitter and receiver. Please note that the Doppler frequency amounts to 6 kHz. It turns out that PRP-OFDM is superior for low SNR values.

We highlight that PRP-OFDM with 8 time domain pilots even perfoms better than CP-OFDM with 8 frequency domain pilots. For higher SNR values the MSE for PRP-OFDM flattens and CP-OFDM with PACE performes better. The error floor occurs because the distortion mainly comes from the useful data rather than from termal noise.

Next it is considered that the two channel estimation schemes behave very different in terms of throughput and power consumption. We follow the argumentation in Section VI and calculate the system capacity with (24) and normalize it on the number of sub-carriers N. Again a relative speed of 30 m/s is assumed and the MSE is calculated as explained above. In Fig. 2 the capacity is plotted for (i) a genuine receiver, i.e. a perfect channel estimation is achieved with completely blind estimation, (ii) PRP-OFDM, and (iii) CP-OFDM with PACE with three different trade-offs. We conclude

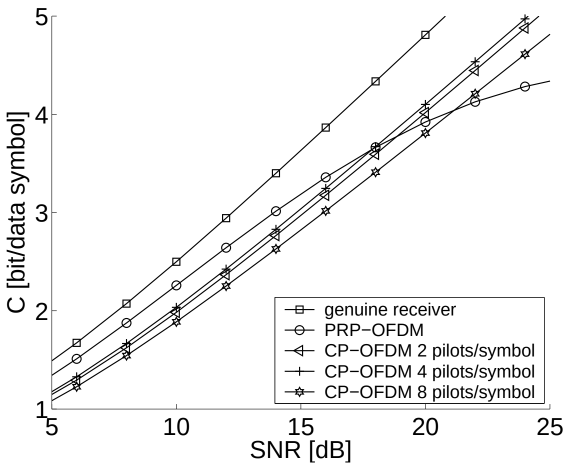

Fig. 2. Total system capacity as a function of the SNR

that for an SNR below about 18 dB PRP-OFDM outperforms CP-OFDM with PACE. Below about 12 dB approximately a 1 dB lower SNR is required to achieve the same capacity as CP-OFDM employing 4 pilots per OFDM symbol. The difference even gets larger for CP-OFDM with 2 and 8 pilots per OFDM symbol. For an increasing SNR the capacity gain becomes smaller for PRP-OFDM and it quickly converges to saturation. That means channel estimation based on PRP-OFDM turns out to be inefficient for high SNR and CP-OFDM with PACE would be the better choice.

It shall be noted that similar results have been obtained with various system configurations, i.e. different bandwidths, different number of sub-carriers, and different channel lengths.

VIII. Conclusion

In this contribution we compared the channel estimation performance of the PRP-OFDM modulation scheme and CP-OFDM in conjunction with PACE. An analysis of the achievable MSE demonstrated the superiority of PRP-OFDM for channel estimation at low SNR. In order to consider the different effective bandwidth and the different power consumption, the overall system capacity was examined. It turned out that for low to medium SNR PRP-OFDM is highly efficient compared to CP-OFDM, while for high SNR the capacity of PRP-OFDM runs into saturation due to inherent interferences. Then the classical CP-OFDM with PACE is the better option. Thus PRP-OFDM is generally limited to configurations with small constellation size and/or powerful error-correcting codes.

Acknoledgement

Note that the BROADWAY project is funded by the Information Society Technologies program of the European Commission (see `http://europa.eu.int/information_society`). The authors would like to acknowledge all the partners of the project for their valuable contributions and discussions: These are Motorola Labs France, Dresden University of Technology, IMST and IRK Germany, TNO FEL Netherlands, INTRACOM and University of Athens Greece and FARRAN Ireland.

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